

MEDDELANDE FRÅN LUNDS ASTRONOMISKA OBSERVATORIUM
Ser. II. Nr. 103.

STUDIES OF IRREGULAR VARIABLE STARS

BY

FRIDA PALMÉR

LUND 1939

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FIL. LIC., LD.

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Preface

The present work has been carried out and completed at the Lund Observatory.

For the observations which are the basis of my determinations of the proper motions of irregular variables I have had the opportunity of using the Repsold meridian-circle of the Observatory.

It is my agreeable duty to express my heartfelt gratitude to Professor Dr. KNUT LUNDMARK, Director of the Observatory, for his kind interest in my work, for much suggestion and criticism, and especially for his kindness in reading my manuscript. My sincere thanks are further due to Dr. LUNDMARK for his recommending me to several grants which have made it possible for me to visit and work at several foreign observatories.

To Professor Dr. WALTER GYLLENBERG, at the Observatory, who has been my teacher from the beginning of my astronomical career, and who has suggested the present subject for my investigation and followed my work with keen interest, I wish to express my warmest gratitude. Dr. GYLLENBERG has introduced me into the theoretical and practical side of observations and their treatment. I am further indebted to him for much suggestion and criticism during the course of this work. Dr. GYLLENBERG has further put his computing assistant at my disposal for several months, and I wish to express my thanks for this help.

To Dr. JOHN OHLSSON, Assistant Professor in Astronomy at Lund, I wish to offer my grateful thanks for valuable discussions on various problems connected with my work, and for reading my manuscript.

To Mr ARTHUR HENRY KING, B. A., I should like to express my thanks for his kindness in improving the language of the present paper.

To Miss ANNA JOHNSON, who has acted as my computing assistant and aid during several periods of my work, I should like to express my great thanks for her conscientious and amiable help.

I should also like to take this opportunity to thank my other colleagues and friends at our Observatory for good fellowship during past years.

It has been my privilege to enjoy several stays at the Observatory of Berlin-Babelsberg. I am indebted to the University of Lund for several grants that have made this possible for me.

To Professor Dr. PAUL GUTHNICK, Director of the Berlin-Babelsberg Observatory, I express my sincere thanks for the hospitality and the facility for work that I have always enjoyed at his Observatory.

To Professor Dr. RICHARD PRAGER, earlier at the Berlin-Babelsberg Observatory, my best thanks are due for placing his very extensive card-catalogue of the literature of variable stars at my disposal during my work at the Observatory, and for his interest in my work and much good advice.

Dr. JULIUS DICK, at the Berlin-Babelsberg Observatory, I should like to thank heartily for his giving me the opportunity to work with the Pistor & Martin's meridian circle, and especially for his making me his temporary assistant in referring the fundamental stars for the photographic AG-observations to the system A_v . The experience obtained in this work has been of great value to me when handling my own observations later on.

To Professor Dr. LEO WENZEL POLLAK, Director of the Geodetical Institute in Prague, I wish to offer my sincere thanks for his hospitality during my stay in Czechoslovakia, and for introducing me into the praxis of harmonical analysis and to the use of the Hollerith-machines for carrying out laborious numerical calculations.

To other colleagues at foreign observatories I wish to express my best thanks for the obligingness and courtesy that I have always enjoyed.

Lund Observatory, October 1938.

Frida Palmér.

Introduction

The present paper is a study on variable stars having complex light-curves, with special regard to results obtained from positions and proper motions of these stars.

The stars investigated here are known in the literature under several names. At different periods they have been called *irregular variables*, *intermediate variables*, *semi-regular variables*, *variables with complex light-curves*, not to mention the more detailed classifications. As will be made clear below, any definition of these variables is bound to take several characteristics into consideration. The above names all refer to the appearance of the light-curves, except that of »intermediate variables». The nature of the curves seems to be best expressed by »variables having complex light-curves», since the essential characteristic of these stars appears to be the existence of several oscillations. Since the name, however, is a long and complicated one, the old term »irregular variables» has been used in the following, when there has been no special need to refer in more detail to the light-variation.

The variables here investigated are characterized by complicated light-curves, which may be the result of an interference of two or several variations with different periods and different or equal amplitudes. From these light-variations arises the characteristic long-period variation which has already been noted by ZESSEWITSCH as a general feature for the so-called μ Cephei-stars. The long-period wave will be present in all variables of this group as a consequence of the interference, and need not have any physical cause of its own. However, only long series of observations can reveal this interference-wave. Therefore only comparatively few stars can at present be classified merely by means of their light-curves. Such a classification would undoubtedly be the most satisfactory one. But for the reasons mentioned it is necessary to have certain simple criteria to recognize an irregular variable by means of other characteristic features, more easily obtained.

The present investigation gives several points which will characterize the irregular variables.

1. The spectra are not limited to one or a few spectral classes. However, the largest number of variables belongs to the classes *K*, *M* and *N*. Most of the irregular variables here considered are consequently red stars.

2. The irregular variables show small range in light variation. The mean value of the amplitude, A , of 580 variables is found to be:

$$\bar{A} = 1^{\text{m}}50 \pm 0^{\text{m}}03,$$

with a dispersion of:

$$\sigma_A = 0^{\text{m}}752 \pm 0^{\text{m}}022.$$

3. The primary periods here denoted as P_1 are distributed over the whole range of periods. But there is a pronounced maximum at 85^{d} and another at 130^{d} . The computed mean value of the period of 224 variables ($P_1 \leq 180^{\text{d}}$) is:

$$\bar{P}_1 = 99^{\text{d}}4 \pm 2^{\text{d}}3,$$

with a dispersion of:

$$\sigma_{P_1} = 35^{\text{d}}2 \pm 1^{\text{d}}7.$$

These data have been used as a criterion for selecting the stars to be included in the present investigation.

In the present discussion irregular variables with very small amplitude ($A \leq 0^{\text{m}}4$) have not been included. Such variables are continually discovered with the aid of the photo-electric cell. However, the data concerning these stars is still very incomplete. Hence it has not been considered justifiable at present to discuss these stars together with the rest of the material, since new discoveries must necessarily still be limited to the brighter stars. Therefore the frequency-curve of the magnitudes will present a peculiar appearance. The stars therefore cannot be mixed until variables with small amplitudes have been found among fainter stars also. Otherwise the fainter magnitudes would have to be left out, which, taking the present limited material into consideration, is not desirable.

Another class of variables, which has not been discussed but might have several features in common with the stars here treated, are the Be-variables.

CHAPTER 1

Early observations of irregular variables

1. General survey of the observations of the variable stars of the 19th century

In the year 1796 the first irregular variable was discovered by WILLIAM HERSCHEL, who noted that α Her showed a small variation in brightness, as it seemed of an irregular kind.¹

The observations of variable stars were continued by JOHN HERSCHEL, who called attention to several variables having no regular period. He predicted the existence of a great number of such variables and attributed the cause of variation to cosmic clouds.

Later on, in his »*Outlines of Astronomy*» (1858), JOHN HERSCHEL discusses the discovery of irregularities in several light-curves of periodic variables, such as α Ceti, particularly at their maximum phase. He points out, however, that these irregularities may possibly be found to be periodic ones.

Via the variables of α Ori-type, at first sight apparently totally irregular, he links up the periodically variable stars with the Novae. He draws this conclusion in the following words: »These irregularities (α Ori-type) prepare us for other phenomena of stellar variation which have hitherto been reduced to no law of periodicity and must be looked upon, in relation to our ignorance and inexperience, as altogether causal. — — — The phenomena we allude to are those of Temporary Stars.»

During the first epoch of the study of the irregular variables these stars were not looked upon as isolated phenomena. The data were too scanty for a separation of the variables into separate groups. Each star was still discussed individually. Since all variable stars previously known showed regular, or almost regular, periodic changes in brightness, it was expected that all of them should do so. Therefore the observers tried to deduce the periods for each new variable. In the case of certain red variables this proved to be extremely difficult. We have already seen that

¹ Phil Trans 86. 166 (1796).

J. HERSCHEL attributed the non-regular variation to cosmic clouds. However, the observers of the middle of the 19th century worked on the supposition that a period existed in each case. They assumed that it might be concealed and hence difficult to elucidate.

The variable stars to-day classified as irregular were recognized as a class of peculiar variables already in the middle of the 19th century. At this time very few variable stars were known, and therefore each separate variable was discussed at length in the astronomical literature. In as much as a large number of variable stars was continually discovered, many of them were found to show identical features in their mode of variation, and hence several defined types of variable stars were recorded. As irregular variables were characterized variables showing an apparent lack of regularity in period and light-variation, in other words variables for which no prediction as to the epochs for maximum or minimum could be made with any degree of accuracy. This fact nearly caused diminished interest in a closer study of these variables. The interest was directed to variables showing regular periods, on account of the greater ease and accuracy with which investigations of these stars could be made.

The most eminent observer during this period was no doubt F. A. W. ARGELANDER in Bonn. A study of the reports of observations made by him and his pupils and other contemporary astronomers, like E. SCHÖNFELD, J. F. SCHMIDT, N. PIGOTT, N. R. POGSON and others, gives a picture of the work on and knowledge about variable stars at that time.

In the time of ARGELANDER and his contemporaries the different types of variables were recognized, but no formal scheme of classification was established. As was remarked above, the variable stars known were still so few in number that each could be, and generally was, discussed separately.

ARGELANDER had the same opinion about the irregular variables as was accepted for variable stars in general, i. e. that the variation was of a periodic nature.² In his reports on observations of variable stars he describes the difficulties when observing certain stars, such as R Sct, α Her, α Cas, α Ori, α Hya — all of which are irregular variables according to the current definition. He relates the irregular occurrence of maxima and minima and the different aspect of the light-curves at different epochs. The periods, i. e. the differences in time between two maxima or two minima, differed from each other when determined at various epochs. All this made predictions of data impossible.

However, to a certain degree irregularities were not unexpected observations, since similar ones were made by ARGELANDER³ in the case of α Ceti, as well as for other stars of long period. It was therefore natural for him to search for periods in cases of stars having apparently irregular variation too. The irregularities occurring in the light-curves of variables having a large amplitude

² See various papers in AN.

³ AN 26.369 No. 624 (1848).

such as α Ceti, were, however, found to possess a certain periodicity. Of course this circumstance interested ARGELANDER and gave him another reason to suppose that the irregular variables concealed in their variation an actual regularity which only long series of observation could disclose. Very illuminating in this respect is his report on α Her, from which we will quote: »Dieser Stern gehört bekanntlich zu den unregelmässigsten unter allen bis jetzt bekannten Veränderlichen. Seine Perioden sind von sehr verschiedenem Dauer, und eben so verschieden sind die Helligkeiten, die er in den einzelnen Maximis und Minimis erreicht. Dabei ist sein Lichtwechsel von Minimum zu Maximum oft so gering, dass er sich sehr schwer von den Beobachtungsfehlern trennen lässt, und nur zahlreiche Vergleichen die Wahren Epochen des Maximums und Minimums mit einiger Sicherheit erkennen lassen.«⁴

This quotation expresses the view on variables of this kind at the time in question. In the literature reports of observations of variables are continually to be found. In these the observers always try to ascertain the periods. Periods were determined from the collected material, but in most cases new data prove to change the previously determined period. In order to show how these variables were regarded, and illustrate the difficulties in these cases, I will cite from another paper of this time. As has already been said, the contemporary of ARGELANDER, SCHMIDT, was also an ardent observer of variable stars. Of α Her he says: »Dieser Stern gehört zu den Veränderlichen, derer Periode grosse Unregelmässigkeiten zeigen. Die Beobachtung seines Lichtwechsels ist mit mancherlei Schwierigkeiten verbunden, und dass diese erheblich sein müssen geht schon aus dem Umstande hervor, dass ungeachtet der seitherigen Bemühungen die Periode nicht als sicher bekannt anzusehen ist. Es wird auch keine konstante Periode existieren, sondern die vorhandene wird den grössten und auffallendsten Schwankungen unterworfen sein.«⁵

From this it appears that SCHMIDT has considered the possibility that these stars do not possess a regular period. From this time on the conception of irregularities grows more and more prominent. A variable for which no regular period can be derived will possibly be noted as an irregular one, and consequently often be dismissed from the subsequent programs of observation.

The human mind certainly reacts against the conception of actual irregularities in nature. There is a tendency to discover a periodicity concealed even where only a total irregularity meets the eye. So will also be the case of the irregular variables. For some time they were accepted as (apparently) irregular, but still, the observers did not abandon their search for periods. Their work has resulted in the disclosure of many periods which at first were concealed in an apparent irregularity. In other cases no periods have as yet been found. It remains to prove whether such variables represent a completely irregular kind of variation or

⁴ AN 44.193 No. 1045 (1856).

⁵ AN 45.61 No. 1060 (1857).

whether they too possess a periodicity of some kind. As ARGELANDER already pointed out, the question depends to a large extent on the completeness and accuracy of the available data.

The preference of the human mind for regularity has by some astronomers been considered a source of observational errors, particularly in the observations of variables exhibiting small amplitude and red colour, both characteristics of the class of stars we are considering. J. SCHMIDT has discussed the difficulty of using white comparison stars when observing a red variable.⁶ In these observations an error is liable to enter on account of the difference in colour between the stars which are compared. He also points out that there is a possibility for apparent periods to arise in the observations from this cause. This calls for the necessity of deriving corrections for such errors.

H. OSTHOFF⁷ is of the opinion that the variations observed in the light of the red stars are to a certain extent only apparent ones, caused by changes in the colour-conception of the observer. To some extent he founds this opinion on a comparison between observations made at the same time by the same observer on different variables. Thus the simultaneous observations by H. E. LAUS of the magnitudes of R Lyr (M6) and g Her (Mb) show a marked parallelism. Likewise the light-curves of α Cas (K0) and μ Cep (Ma), deduced from simultaneous estimates by J. PLASSMANN, agree remarkably well. According to OSTHOFF the perception can vary with a short as well as with a long period.⁸

There is no doubt that the observations of variables with small amplitudes must be accepted with criticism and care. It is also necessary not to overlook the possibility of the presence of false periods. But still, the reality of the variation in the light of these stars cannot be doubted, on account of the large material which is available to-day. LUDENDORFF⁹ points out that the reality of the variation is clearly shown from the fact that regular periods have been discovered for many variables, which were previously believed to be entirely aperiodic. Furthermore, the selenium cell, which is sensitive to red light, has with small errors of observation proved the reality of variations in the brightness of these stars. Such phenomena as physiological changes in the observers' conception may, just in cases of small amplitudes, be more important than at first sight might be considered. The complicated light-curves might in some cases originate from a fact like the one mentioned. It would certainly be of great interest to separate the physiological effect from the actual variation in the star-light. This effect would, however, appear only in magnitudes obtained through direct estimates, and should be eliminated in the cases where the magnitudes are determined photometrically or with a photo-electric cell. But direct estimates by trained observers are more

⁶ AN 45.129 No. 1065 (1857).

⁷ AN 212.97 (1920).

⁸ HdA VI. 2 p. 167 (1927).

⁹ HdA VI. 2 (1928).

easily performed, and observations of this kind cannot be disregarded, even at the risk that personal errors enter.

Another opinion about the red stars deserves to be mentioned here, viz. that of T. W. BACKHOUSE¹⁰ that *all red stars*, especially all N-stars, are intrinsically variable.

2. Classifications of variable stars

From time to time a number of different classifications of variable stars have been presented. It is not my intention to discuss them all, since I am not going to deal with the variable stars as a whole but only with a certain part of them. The purpose of the present chapter is to illustrate the relation between the irregular variables and other classes of variables.

The first formal classification of the variable stars, based on twelve variables, was presented by PIGOTT¹¹ in the year 1786. His classification contains the following three groups:

- I. Long-period variables.
- II. New stars.
- III. Short-period variables.

At this time the irregular variables were not yet acknowledged.

A second classification was made by OLBERS¹² in the year 1816. Irregular variables were still not distinguished as a separate class. Two irregular variable stars in the list of OLBERS were classified as short-period variables.

H. J. KLEIN¹³ presented a third classification in the year 1874. Here the irregular variables were for the first time formally recognized as a special class. His classification distinguishes the following four classes:

- α) Variables with non-periodical light-variation,
- β) Variables with long and very irregular periods,
- γ) Variables with moderate and very regular length of period,
- δ) Variables with rapid change of light, confined to a few hours.

The next classification was made by E. C. PICKERING¹⁴ in the year 1881. Here group III includes the irregular variables, with the following definition: »Variables of small range or irregular variation according to laws as yet unknown».

The change in the brightness of this type of variables was found impossible or at least very difficult to predict, since no period was perceptible. The range was found to be small. It was, however, noticed that in some cases an increasing number of observations revealed a period in the light-variation for stars which were earlier supposed to be quite irregular.

¹⁰ JBAA 27. p. 382 (1913).

¹¹ Phil Trans 76.214 (1786).

¹² Zeitschrift 2 181—3 (1816).

¹³ Sirius 6.278 (1874).

¹⁴ Proc Amer Acad 16 (1881); HA 48.94 (1903).

When the material of observations had increased, it was gradually proved that a comparison between the individual light curves of the irregular variables presented essential differences in the change of light, as well as in periods and in spectral class. Hence it was necessary to divide these variables into smaller groups, which with increasing data and extended experience were found to differ fundamentally.

In PICKERING'S second classification of 1911 such a distinction was introduced for the first time.¹⁵ His classes II b, II c and III contain the irregular variables, with a separation into U Gem-stars (II b), R CrB-stars (II c) and irregular variables such as α Ori and R Sct (III). With these three groups we have in fact reached a modern view of these stars.

The new classes of irregular variables mentioned above are found to have slight variations in later classifications by S. D. TOWNLEY,¹⁶ A. A. NIJLAND¹⁷ and in its main features also in the classification by P. GUTHNICK.¹⁸ Here the groups II, III, IV and VI represent all stars with irregular change of brightness. Groups IV and VI, including e. g. μ Cep and R Sge, correspond to PICKERING'S group III.

A classification by K. GRAFF¹⁹ differs only in minor details from that by GUTHNICK.

Since this time the classification given by H. LUDENDORFF²⁰ has been the one most used. With regard to the irregular variables it may be remarked that his classes II, III, IV, VI and VII cover all stars now recognized as irregular.

II. Nova-like stars:	T Pyx, η Car
III. R CrB stars:	R CrB, RY Sgr, SU Tau
IV. U Gem stars:	U Gem, SS Aur, SS Cyg
VI. μ Cep stars:	μ Cep, ϱ Per
VII. RV Tau stars:	RV Tau, R Sct, R Sge.

For further detailed studies of the classification mentioned for all types of variables cf. the extensive treatment by H. LUDENDORFF in *Handbuch der Astrophysik* Vol VI, Part 2.

A recent classification has been presented by K. LUNDMARK,²¹ which classification is made with regard to the mode of light-variation. The following classes have been distinguished:

- I. Short-period variables.
- II. Long-period variables.
- III. Cyclical or semi-cyclical variables.
- IV. Irregular variables.

¹⁵ HC 166 (1911).

¹⁶ Publ ASP 25 p. 239 (1913).

¹⁷ Hemel en Dampkring 10 p. 184 (1913).

¹⁸ Die Kultur der Gegenwart III Band: Astronomie p. 435 (1921).

¹⁹ SCHEINER-GRAFF: Astrophysik p. 386 (1922).

²⁰ Probleme der Astronomie. Festschrift für *Hugo von Seeliger* (1924).

²¹ Lund Medd Ser II No. 74 (1935).

Class I has the following subdivisions:

- 1) the short-period Cepheids or the cluster variables with P shorter than 1^d2 ,
- 2) the classical Cepheids having P between 1^d and 10^d ,
- 3) the long-period Cepheids with P between 10^d and 127^d , and the RV Tauri variables and the red Cepheids.

The Nova-like stars have been classified with Novae.

Considering the present knowledge of data, it would seem preferable to combine classes III and IV into one class, which may be called »irregular variables». Further, variables of class I, 3 with periods larger than 50^d and the RV Tauri variables may be considered also to belong to this class. The latter stars are possibly most correctly to be regarded as transitional cases between Cepheids and irregular variables.

3. Studies of irregular variables as a separate class

From now on the study of the irregular variables enters a new period. For some time all these stars were denominated as irregular and were classed as one group. However, it gradually became clear that that was not correct. Already in my preliminary investigation on these stars²² I have excluded groups II and III from a general statistical discussion. G. R. MICZAIKA has subsequently done so too. The R CrB-stars show clearly a variation of quite another kind than the μ Cep-variables and should not be included in the same investigation. The Nova-like variables have by LUNDMARK been included in the Nova-group, since intrinsically they do not differ from these stars. The U Gem-stars also are probably a class different from the irregular variables in the present meaning. There remain the μ Cep-stars and the RV Tau-stars. Opinion is not as yet quite settled about the latter stars. I will return to them in a later chapter.

We have thereby arrived at a stage in the history of the irregular variables, where only classes VI and VII in LUDENDORFF's scheme, i. e. the μ Cep-stars and possibly the RV Tau-stars, are classified as irregular variables.

For a long time only individual investigations of irregular variables were undertaken. In most of them the observers try to establish periods for the light-variations. In the literature we might find, if we scrutinize e. g. the »Geschichte und Litteratur der veränderlichen Sterne», values of periods given together with an additional remark concerning irregularity. Often previously obtained periods are cancelled and the classification »irregular» has been put instead. Sometimes it happens the other way round. In the brief collections of the result of observations we are able to grasp something of the difficulties which the observers have had — and still have — to overcome. We also arrive at the conclusion that

²² LOC No. 3 (1931).

irregularity in the variation of light is a much more general occurrence than might at first be supposed. Besides, stars classified as irregular but combined with period-values have escaped further treatment as irregular because their periods have been given. The period once given, these stars have been considered as *bona-fide* long-period variables. If we go through the available literature of the suspected stars one by one we shall find irregularities noted in the observations of most of the stars of spectra M and N without emission-lines in connection with the small amplitude. These stars have earlier been included in the list of long-period variables.

If we examine the notes of irregularity, it will be evident that irregularities are mostly observed in the light-curves between 50^d and 130^d. This interval may be a critical zone, where irregularities easily arise. In the frequency-curve of periods there is a gap (or least a minimum) between the Cepheids and the long-period variables for the above mentioned interval in period. The irregular variables might just fill in this gap in the general frequency-curve and thus complete the series of variables.

* *

The investigations of the irregular variables show a new aspect with the discussion by GERASIMOVICH in the year 1927 of the variable Z Leo.²³ Here we have a variable of spectrum Mb, which is proved to show a well-defined period but is still subject to irregularities in the maxima and minima of its light-variation. The maxima show larger oscillations than the minima, a fact which may prove to be of some importance in coming discussion.

In the course of his work a new characteristic was introduced, viz. that of the »cycle» of a variable. He calls attention to the fact that many variables with red spectra possess a certain regularity in their light-variation. Earlier this variation had been supposed to run without periodicity. This regularity could not exactly be described as a period, because the very definition of a period assumes a constant recurrence of the phase in question. In the present case, for instance, a maximum does not reappear after a constant interval of time, as is the case of the Cepheids or long-period variables. The maximum in our case is more or less irregularly distributed in time. Still, a certain phase-interval may recur more often than others. Thus we shall have a frequency-maximum for this most frequent interval of time. For these frequency maxima GERASIMOVICH introduced the name »cycles». The cycle of an irregular variable will prove to be of the same importance as the period for a regular one.

The name *semi-regular variables* has been assigned to variables the light-variation of which can be characterized by a cycle, and to variables in general with a certain periodicity combined with irregular changes.²⁴

²³ HB 849 (1927).

²⁴ The term used in the German literature is »halbregelmässige Veränderliche».

Subsequent to this work great interest has been taken in the semi-regular variables. The members of this group have been investigated individually, as it is necessary to obtain more detailed data concerning their light-variation. In the course of this work many interesting features have been brought to light.²⁵

GERASIMOVICH has made the first general discussion of the semi-regular variables in HC 341 and 342. The first paper treats the RV Tau variables, and the second the variables of the irregular group. It should be noted that the RV Tau variables have been considered to belong to the intermediate group.

In the paper mentioned GERASIMOVICH has given a new and more detailed classification than earlier ones of the irregular variables. This classification is to a great extent founded on the principle that the stars shall satisfy the statistical relation between period and spectrum first worked out by SHAPLEY.²⁶ The stars which satisfy this relation are (by GERASIMOVICH) said to belong to the »principal sequence». The cluster-type variables, the Cepheids, the RV Tau-variables and the long-period variables thus belong to the principal sequence.

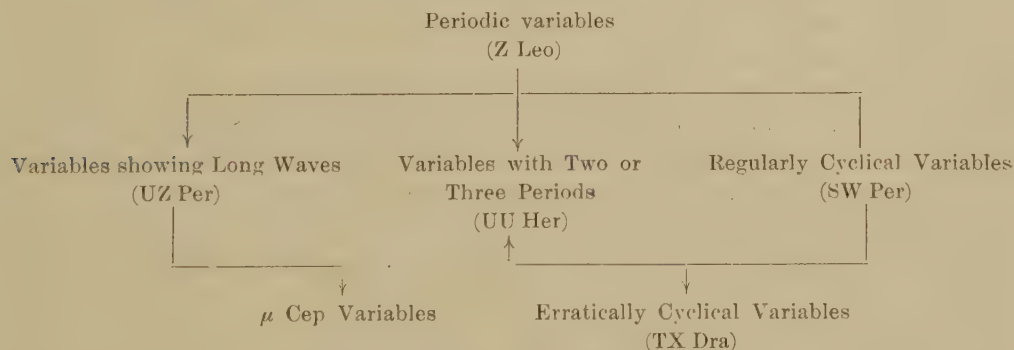
GERASIMOVICH further distinguishes the following two principal classes of intermediate variables:

- A. Stars belonging to the principal sequence,
- B. Stars *not* belonging to the principal sequence.

Group A includes variables with known period and fairly regular light-curves. Variables belonging to this group have previously been included among the long-period variables. They have, however, earlier spectra than the latter stars, and are somewhat similar to the Cepheids with regard to range and shape of the light-curve. The last group B is characterized by spectra which, considering their mean period, and from the point of view of the period-spectrum relation, are too red. These two principal groups contain the following sub-groups:

- A. 1. RV Tauri variables.
- 2. Variables with periods of about 100^d.

For group B we copy here the author's own scheme:



²⁵ See various Harvard Circulars and Harvard Bulletins.

²⁶ HB 861 (1928).

With increasing observational material the attitude to the intermediate variables will change. It is soon evident that this kind of variation is not limited to the small interval of periods from 50^d to 100^d , though irregularity seems to be favoured in this interval. On the contrary it seems to be a more general occurrence than might at first be supposed. I will return to this point in more detail in a later chapter.

Another modern classification of the intermediate variables is given by L. JACCHIA in his handbook,²⁷ where the following classes are distinguished:

a) AF Cygni variables.

The periods of this class of stars are distributed over an interval between 50^d and 400^d . Only one variable (W Per) has a period larger than 400^d , i. e. 496^d . The appearance of the light-curve often changes from one period to another. In many cases the light-curves show »RV Tau-anomalies». The fact that their mean light varies with a long period is also characteristic of these stars. Thus TW Peg has a second period of 934^d , and UZ Per one of 920^d .

b) Variables with several non-simultaneous periods.

Variables belonging to this class present in their light-curves more than one period of different length, and these periods do not overlap. Each of them represents the variation of brightness within a certain interval of time. Then an interval with purely irregular variation follows, after which the second period follows. Only very few representatives of this class are known. JACCHIA has mentioned the two following:

V Lyn with the periods $87^d.1$, $54^d.2$, $163^d.4$.
 UU Her » » » 72.1 , 45.2 .

c) Variables of μ Cep-type.

This class represents the variables earlier supposed to be totally irregular. But their variation is still not without a certain regularity. These stars ought rather to be described as variables having irregular variations of a shorter period overlapping long waves. Thus the stars μ Cep and α Ori earlier regarded as purely irregular have disclosed three periods in their light-curves. And the same is the case with other irregular variables.

When examining curves representing the change in brightness for irregular variables — to use this term generally — one is led to the opinion that there cannot be any real difference between the variables of AF Cygni-class and the μ Cephei-variables or the sub-classes of GERASIMOVİČ's scheme.

²⁷ Pubblicazioni dell'Osservatorio astronomico della R. Università di Bologna Vol II No. 14 (1933).

CHAPTER 2

The periods of the irregular variables

The historical notes in the previous chapter show some phases of the development of our knowledge as to the irregular variables. These stars were at first assumed to change their brightness periodically, but the failure to confirm a period, once established, led to the assumption that they did not undergo regular periodic changes. Later on, the statistical periods, or cycles, were introduced as a characteristic, in several ways comparable to a period. Finally, recent investigations have proved these variables to possess a complicated periodicity in their light-variation.

I. The frequency-curve of the periods

For 281 stars of the material available in the present compilation it has been possible to determine a period of variation. If the material is restricted to embrace stars with known spectrum, the number of periods is reduced to 176.

It has been pointed out earlier that the periods of the irregular variables are not invariable quantities like those of the Cepheids and long-period variables. The periods given in the present list of variables must be regarded as relating to a mean period. This will be explained more clearly in the subsequent discussion.

Besides a primary period, the irregular variables have been found also to possess a long period.

In the following investigations the short primary mean period, denoted P_1 , has been considered. The period P_1 may be regarded as the original period of variation, intrinsically related to the star itself. Mistakes in noting the periods may have occurred, especially among the larger values of P_1 , since these might in some cases represent a second period in the variation because the primary period has as yet escaped detection. However, such errors are statistically of small importance.

Figure 1 shows the frequency-curve of the periods of the irregular variables. It is of interest to observe that, though a very pronounced maximum is present around 80^d — 100^d , the whole range of periods is represented. The shorter periods

are cut out owing to the necessary choice of a limit. The irregular variability therefore seems to be a universal occurrence in the series of variable stars, though the periods of about 100^d in stars with M- and N-spectra seem to favour this kind of variation.

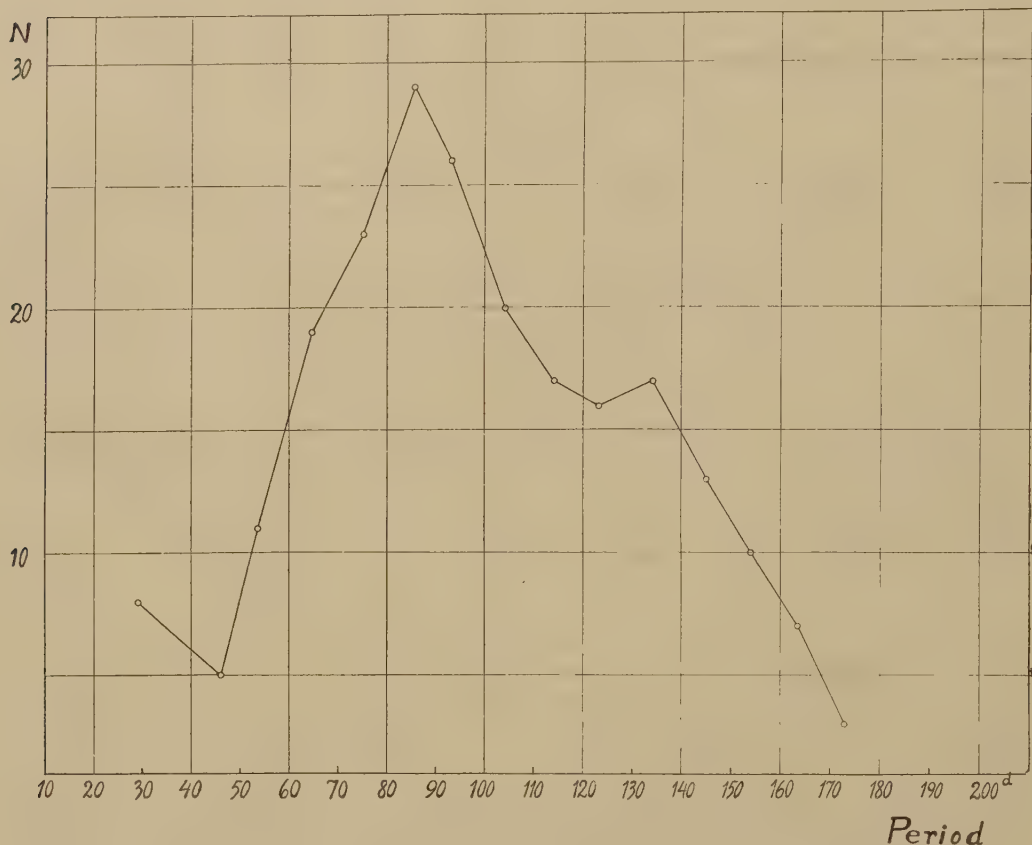


Fig. 1. Distribution of the periods of the irregular variables. Periods larger than 180^d left out.

The appearance of the frequency-curve seems to indicate the existence of *two* different groups of periods, one with the maximum at about 85^d , the other at about 130^d . Whether the two maxima correspond to a real difference or are caused by statistical selection is not possible to say on the basis of the present material.

It must be remembered that our material of variable stars is still very incomplete and often uncertain. Therefore the present calculations and conclusions on the basis of the periods must be regarded as a first attempt to range them among the series of variables. However, since the problem of these stars' following the same general laws as the Cepheids or long-period variables is of very great interest, the statistical treatment below has been thought to be justified.

In the present investigations the periods larger than 180^d have been excluded, owing to the incompleteness of the material of larger periods.

After this exclusion there remain 225 variables with known periods. Of them a number of 128 stars have spectral classification.

The computed mean value of the periods less than 180^d was 99.4 ± 2.3 ($N=225$) with a dispersion of 35.2 ± 7.7 .

It is also of interest to note that the mean value of the periods is essentially the same as indicated by the preliminary investigations. It is therefore probable that the maximum of the curve will change inconsiderably with increasing material. Thus the favourable periods for irregular variation seem with great probability to range from about 70^d to about 140^d .

II. The complex light-curve and the multiple periods

1. Earlier work.

In my cited preliminary work ¹ on the semi-regular variables, the mean absolute magnitude of these stars, calculated from 90 proper motions, was entered into the general period-luminosity diagram obtained by GYLLENBERG.² The absolute magnitude was found to correspond to *two* different periods: one of about 90^d , the other of 600^d or longer. The observations showed the first of these values to correspond fairly well to the mean value of the available intermediate periods. It is further of interest to observe that this mean period is situated at the point on the general frequency-curve of periods where a minimum occurs. The value obtained may prove the semi-regular variables to represent a connecting link between the Cepheids and the long-period variables, as has been supposed by several investigators.

The explanation of the existence of a secondary period was not clear at first. Does it mean that these variables have *either* a short *or* a long period — longer than a period of any long-period variable — or that a star has *both periods* simultaneously? A study of the literature available at that time shows the latter alternative to be in accordance with the observations. Certain variables were known to have their variation in brightness represented by two periods, very similar in length to the statistical mean values derived above. The most prominent representatives at that time were UZ Per and TW Peg. The first star shows a short period of 90^d , a long one of 920^d , and a third supposed period of 5000^d (Sp. M4, $A=1.7$). TW Peg has a short period of 90^d and a long one of 934^d (Sp. M8, $A=1.7$).

The history of UZ Per is very typical for a variable of the irregular type. At first the variation in brightness was considered not to exhibit any regularity.

¹ LOC No. 3 (1931).

² Lund Medd Ser II No. 54 (1930).

Then the possibility of an RV Tau-variation was suggested. Later on the presence of a long period was suggested, and the star was classified as a long-period variable. When still more extensive series of observations were available, the double periodicity was discovered. The case of TW Peg is similar.

Further data of derivation of double periods have been given in several Harvard Bulletins, in the Russian Journal »Veränderliche Sterne» and in other papers.

The presence of two periods in the light-curves of certain variables of this type was, however, noticed already in the middle of the previous century. In the year 1864 we find a remark to this effect by J. SCHMIDT³ in the case of γ Her. In his observations of the brightness of this star SCHMIDT found an indication of a long period of two years and one month together with a secondary smaller period, the length of which varied between the limits 48^d to 106^d. The observation was accepted merely as a curious fact.

The existing observational material was at that time too scanty to allow a general discussion of the irregular class as a whole. In most cases very long series of observations are needed to prove the existence of a long wave in the light-variation.

2. Conclusions based on the present material.

In order to make a survey of the problem of the periods of the intermediate variables, as many available light-curves as possible were collected from various sources, photographed, and enlarged. Most material was obtained from L. JACCHIA's collection of observational data and light-curves,⁴ from which he has also determined a great number of periods. Further material was obtained from various Harvard Bulletins, Harvard Annals and Russian Astronomical Journals. In all 63 light-curves were brought together.

In some cases new light-curves were constructed by the present author. For this purpose series of observations from Journal des Observateurs and Harvard Bulletins were used. The observations were referred to a uniform photometrical system, which made it possible to use the original magnitudes without corrections.

Discussion of the light-curves.

Characteristic for the light-curves of the intermediate variables are the great individual differences in the curves of different stars. In the case of the Cepheids and the long-period variables a fairly strong resemblance is shown from one star to another.

³ AN 62 (1467) p. 39 (1864).

⁴ Bologna Bull Vol II No. 14 (1933).

The light-curves, though individually different, show a general similarity.

A survey of a good many curves makes it evident that some are more regular than others. Therefore many variables of this type are counted as regular and entered in the group of the long-period variables.

The regularity in the variation of light, as seen from the light-curves, might be described as follows.

The apparently irregular appearance of the curve might be explained as the result of a very complicated variation, which might be caused by the interference of two or several original oscillations, which with great probability can be supposed to have regular periods. The period P_1 discussed above might be considered as a mean value of these original periods. An accurate analysis of the light-curves with a deduction of the original periods could only be carried out by means of a treatment of the observations by the methods of harmonic analysis. Investigations of this kind will, however, require long, uninterrupted series of observations. At present there would be only very few stars suitable for detailed investigations. Such investigations were planned by the present author, but for the reasons mentioned must be set aside.

In single cases the method of harmonic analysis has been used to ascertain the period of variation. The variable R Sct has thus been subjected to such a treatment by K. STUMPF,⁵ and RV Tau has been investigated in a similar way by J. VAN DER BILT.⁶

At present we shall refer only to what can be learned from a direct study of the light-curves and periods.

The general appearance of the light-curves of the irregular variables can be described as a periodic variation P_1' with a secondary variation P_1'' , observed e.g. as a secondary maximum, progressing relative to the first period along the time-axis. Further, the total amplitude varies from a minimum near zero up to a maximum value. The variation in range thus defines a long period, P_2 , in the light-variation.

The light-curves of the irregular variables, as we have found from the present material, are subject to several dissimilarities. These are perhaps best expressed in the above mentioned long-period variation in amplitude. When the present material of light-curves and periods is considered, the presence of a long wave seems to be a characteristic feature for variables of this group. This has already been pointed out by ZESSEWITSCH⁷ in connexion with his investigation of μ Cep.

Taking the extreme types of the light-curves into consideration, a formal classification of the light-curves of the irregular variables may be made along the following main lines, when special attention has been paid to the long waves:

⁵ AN 253. 110 No. 6053—54 (1934).

⁶ Rech Astr de l'Obs d'Utrecht 6.94 (1916).

⁷ See Mirov Bull 22; AN 235.181 (1929); Leningrad Bull No. 2 (1933).

- A. A variation in amplitude with or without a variation in the mean brightness.
Ex. AF Cyg, RY Dra, TX Dra, X Her, V Lyn,
B. Short-period variations appearing as details on a long-period wave. Ex.
 α Ori, μ Cep, VY Cas, RR CrB.

Remark:

There should also be noted as a special case the variation in mean magnitude with alternatively constant minimum phase, when the maxima are subject to long-period fluctuations (Ex. T Cen).

The period P_2 may originate from a series of events taking place in the star itself, independent of the waves of a short period P_1 . On the other hand, it may be the result of an interference of two (or more) original short-period waves P_1' , P_1'' , etc. If this were the case, then the long-period variation in the mean light would be a formal occurrence, which had no direct cause in the star itself. We might regard the two groups of variations from this point of view.

Assuming the primary variations P_1 in the brightness to be represented by two simple sine-functions, it is possible to produce a series of curves as a result of the interference of the two mentioned waves. These will correspond in their main features to the observed curves of the irregular variables. The interference of the two periods will cause the appearance of a long period P_2 in the amplitude. The length of this »period of interference« will depend mainly on the difference in length of the two original periods.

Graphical representation of the interference of periods.

It will be shown that it is possible to produce by means of a simple process of interference the two extreme types of irregular variables represented by the classes A (AF Cygni) and B (μ Cep, α Ori). Consequently all other types of light-curves between these extremes can also be produced.

Of course, there may be more than two primary variations in the light of irregular variables. In many cases the complexity of the curves suggests the presence of several original periods.

However, at present we will consider the result of the interference of two variations, which may be written in the following way:

$$A' = A_0' \sin \frac{2\pi t}{P_1'}, \quad A'' = A_0'' \sin \frac{2\pi t}{P_1''}. \quad (1)$$

A' and A'' are the amplitudes at the epoch t ; and A_0' and A_0'' are the original amplitudes.

P_1' and P_1'' are the periods in the two primary variations respectively.

1. For the sake of simplicity A_0' may be supposed to be equal to A_0'' and put equal to A_0 .

An interference of the two waves in question will result in an amplitude A which is given by the following expression:

$$A = A_0 \left(\sin \frac{2\pi}{P_1'} t + \sin \frac{2\pi}{P_1''} t \right). \quad (2)$$

This sine-function is easily transformed into the following form:

$$A = 2A_0 \cos \left(\pi \frac{P_1'' - P_1'}{P_1'' P_1'} t \right) \sin \left(2\pi \frac{P_1'' + P_1'}{2P_1'' P_1'} t \right). \quad (2a)$$

From equation (2a) it is evident that the total amplitude A is expressed by a new sine-wave having the argument:

$$\left(2\pi \frac{P_1'' + P_1'}{2P_1'' P_1'} t \right).$$

The period in the amplitude caused by the interference is found from the expression:

$$t = \frac{P_1'' P_1'}{P_1'' - P_1'}. \quad (3)$$

Thus the period of interference, that is the long period in the variation, will increase with decreasing difference between P_1'' and P_1' . It is thus possible to construct theoretical light-curves, corresponding to the actually observed curves.

The above expressions (2) and (2a) for the amplitude of interference may be transformed to functions of the frequency ν instead of the period.

Then we have:

$$A = A_0 (\sin \nu_1' t + \sin \nu_1'' t), \quad (4)$$

and:

$$A = 2A_0 \cos \left(\frac{\nu_1' - \nu_1''}{2} t \right) \sin \left(\frac{\nu_1' + \nu_1''}{2} t \right). \quad (4a)$$

Here the difference between the frequencies will define the length of the period of interference.

The length of the original periods and the difference between them may depend on the physical nature of the star. Possibly the value of the ratio p_1 between the long period P_2 and mean original period P_1 , i.e. the difference between P_1' and P_1'' , is an expression for a certain physical quality of the star. The run in the values of p_1 with spectral class (see p. 000) might possibly express some special physical peculiarities in stars of different spectral classes. Since, however, the material is at present very insufficient, we will not enter upon further discussion of this matter.

2. We assume: $A' \neq A''$.

In this case the total amplitude A has to be calculated by use of the expression:

$$A = A_0' \sin \frac{2\pi}{P_1'} t + A_0'' \sin \frac{2\pi}{P_1''} t. \quad (5)$$

This function will certainly give the most correct description of the actual results of the interference, since our assumption of equal original amplitudes must be regarded as a too special case.

The difference in the underlying amplitudes will make the light-curve more complicated. The importance of considering the different amplitudes will be shown in the reconstructions of the extreme class B (p. 26).

The figures 2—6 show the result of the attempts to construct the different types of light-curves of irregular variables made by the present author.

It has been assumed by the present author that all light-curves are results of interferences of two or more primary periods $P_1^{(i)}$. The difference between the primary periods P_1' , P_1'' etc. varies. Thus it is possible to obtain light-curves with different lengths of their long wave. It is of importance to note that all long waves may be considered as the result of the interference. Thus they need not have any physical cause in the star itself.

We may regard the two apparently extreme classes of curve: A and B .

The figures 2—4 represent class A . This type of curves is obtained by taking the periods P_1' , P_1'' to have values not differing very much. Then we find a resulting curve with a long period in the amplitude. This period depends on the difference between P_1' and P_1'' in accordance with equation (2 a). This class seems to be the one most frequently represented in the material to hand.

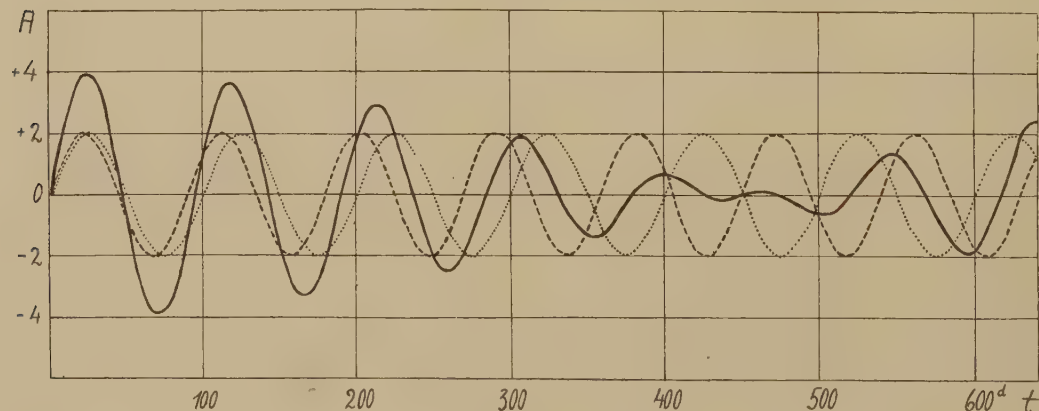


Fig. 2.

$$A = \sin \frac{2\pi t}{90} + \sin \frac{2\pi t}{100} \quad [A_0 = 2].$$

Total curve ———
 $P_1' = 90^d$ - - - - -
 $P_1'' = 100$

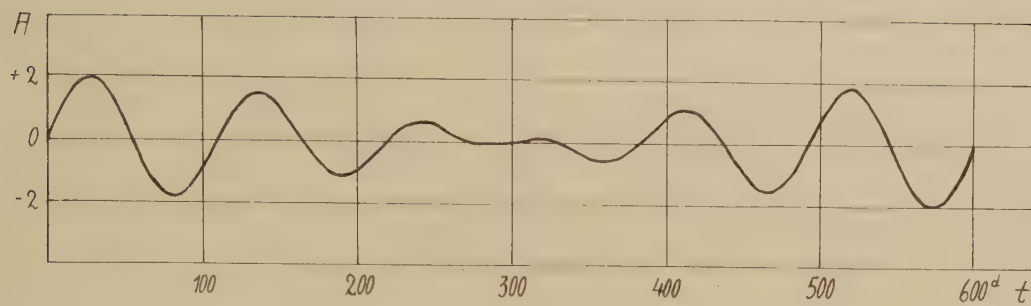


Fig. 3.

$$H = \sin \frac{2\pi t}{100} + \sin \frac{2\pi t}{120}. \quad [A_0 = 2].$$

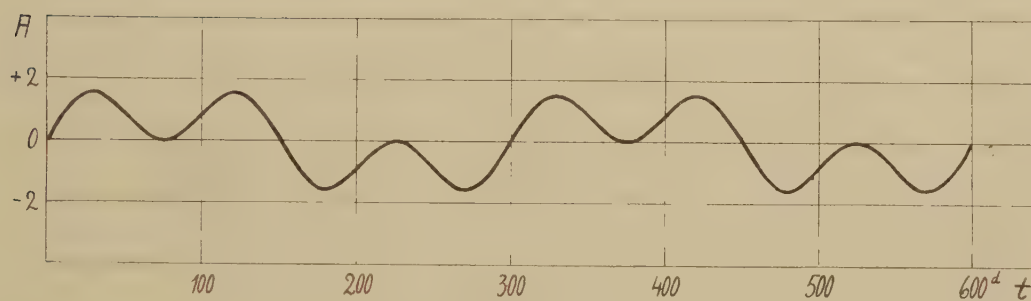


Fig. 4.

$$H = \sin \frac{2\pi t}{100} + \sin \frac{2\pi t}{300}. \quad [A_0 = 1].$$

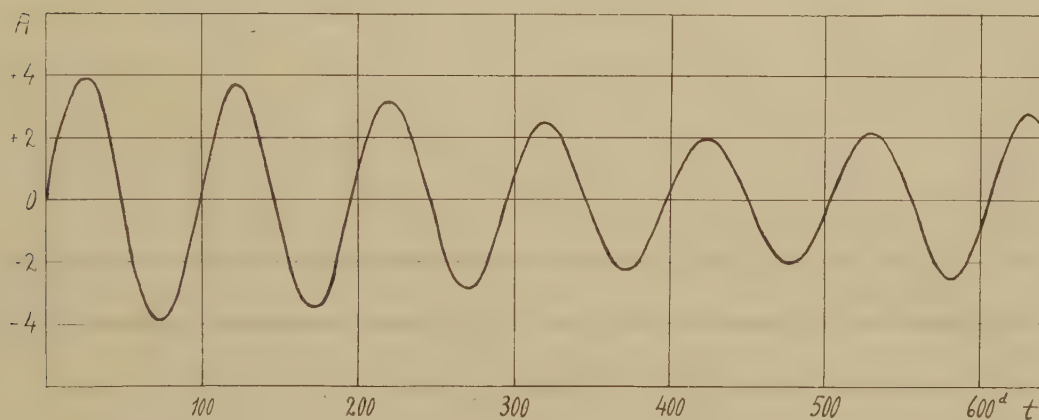


Fig. 5.

$$H = \sin \frac{2\pi t}{90} + 3 \sin \frac{2\pi t}{100}. \quad [A_0 = 1].$$

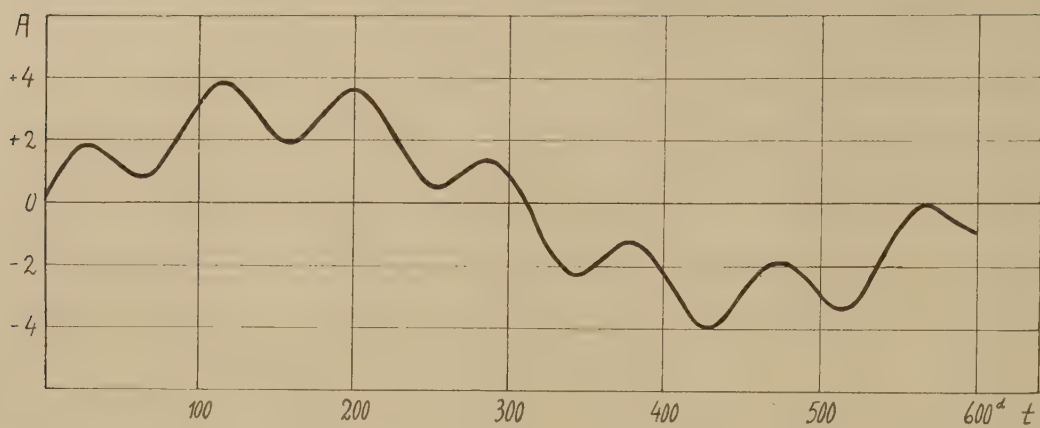


Fig. 6.

$$A = \sin \frac{2\pi t}{90} + 3 \sin \frac{2\pi t}{600}. \quad [A_0 = 1].$$

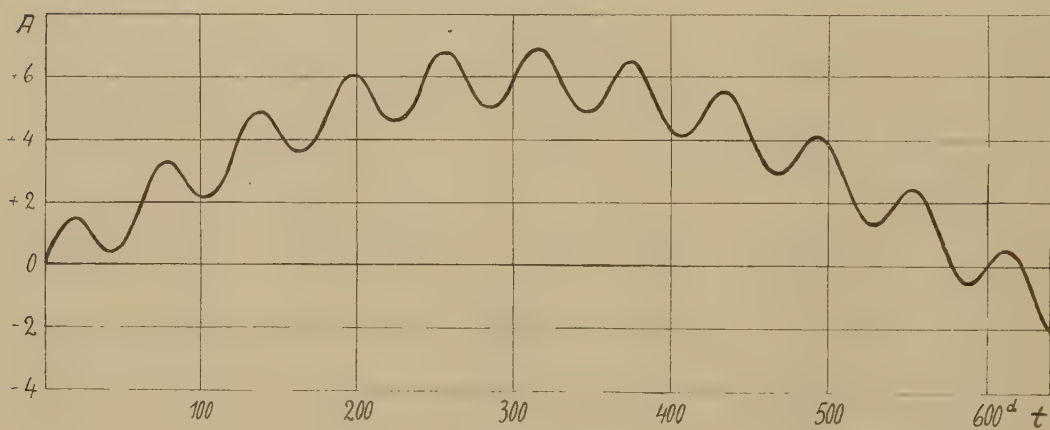


Fig. 7.

$$A = \sin \frac{2\pi t}{60} + 6 \sin \frac{2\pi t}{1200}. \quad [A_0 = 1].$$

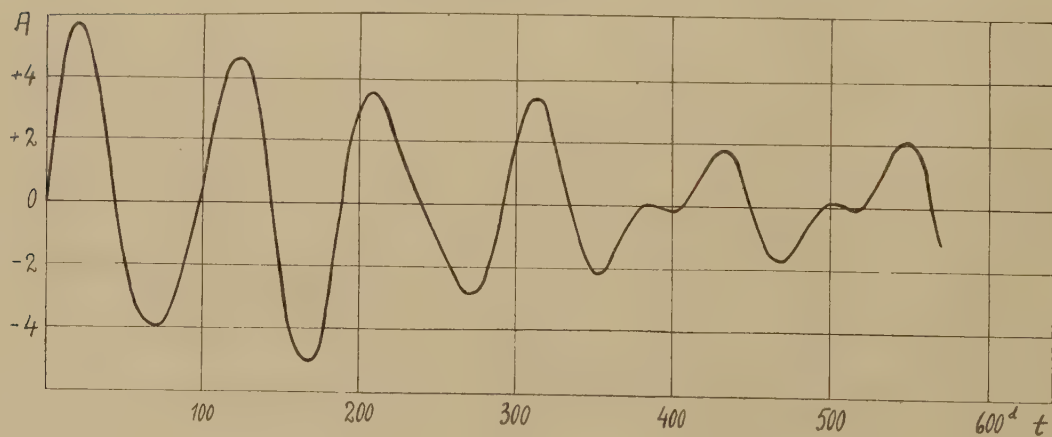


Fig. 8.

$$A = \sin \frac{2\pi t}{60} + 2 \sin \frac{2\pi t}{90} + 3 \sin \frac{2\pi t}{100}. \quad [A_0 = 1].$$

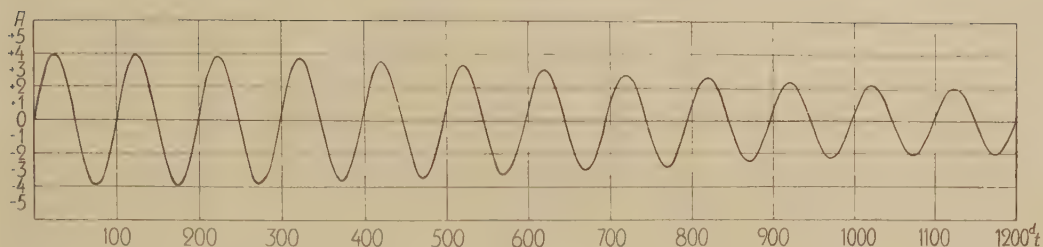


Fig. 9.

$$A = \sin \frac{2\pi t}{96} + 3 \sin \frac{2\pi t}{100} \quad [A_0 = 1].$$

Class *B*, illustrated by figures 5—7, may be constructed in the following way. P_1' is taken as very small, in comparison with P_1'' . Further, A' must be essentially smaller than A'' . In this way the characteristic curve of α Ori, μ Cep and other stars has been obtained. In these light-curves a longer period P_2 is to be expected, and it has also been found.

As examples may be mentioned: —

α Ori: STEBBINS: selen. phot. 1908—1912:

- 1) long variations: amplitude $0^m.5$
- 2) 250^d » 0.2.

GUTHNICK-PRAGER: photo-electrical measures:

- 1) long variations: large amplitude
- 2) short » (11^d): » $0^m.1$
- 3) period of 6 years.

μ Cep: ZESSEWITSCH: 1916—1928:

- 1) 100^d amplitude $0^m.1$
- 2) 600 » 0.7
- 3) 13 years » 0.5.

ZESSEWITSCH:

New investigation based on observations 1915—1929:

- 1) 91^d
- 2) 750
- 3) 13 years.

ZVEREV:

- 1) 40^d—100^d
- 2) 900.

It must be pointed out that in class *B* the observed long-period wave, that is the period of interference, differs by a very small amount from the *original* period P_1'' . (See Fig. 6.) This is the essential difference between classes *A* and *B*,

and of course it is a purely formal difference. In fact there is no actual difference between the two classes. They are the results of a phenomenon of interference.

Between the extreme classes of curves there are all possible variations to be observed. The existence of several original periods with different amplitudes is to be regarded as the most normal occurrence. This will account for the very complicated observed light-curves found in several cases.

It is also of interest to observe the RV Tau-phenomenon in the light-curves with certain values of the original periods. (See e.g. Fig. 5 with $P_1' = 100^d$, $P_1'' = 300^d$.)

The observed light-curves of irregular variables, however, do not show the smooth appearance of the theoretically constructed curves of interference. This is, of course, to be expected. The correspondence might be closer, if a more complicated function were used to describe the original variation, or if several waves were allowed to interfere. But the main thing is that the present attempts prove it possible to obtain a type of curve characteristic of the irregular variables by means of a comparatively simple process of interference.

The observed curves generally show a smoother minimum than maximum phase. The latter is often found to show details of irregularity, such as secondary maxima. This might be explained from the greater difficulties in observing the stars at minima. The uncertainties in the observations, and consequently the larger mean errors, would tend to smooth out the details. On the other hand the difference in the appearance might have a real correspondence in the physical conditions of the stars. These variables, when not in minimum, are possibly in labile states, where fluctuations should be favoured.

3. Statistical investigations into the multiple variation.

The complex light-curve, or the curve with multiple periods, has been the object of discussion by W. ZESSEWITSCH,⁸ K. O'CONNEL,⁹ J. VAN DER BILT¹⁰ and others.

ZESSEWITSCH has investigated the correlation between the short and the long periods of nine variables with double period. Plotting the ratios p between the long and the short period against the logarithm of the short period, he formed the following connexion:

$$p = 80.2 - 44.5 \log P + 6.7 (\log P)^2. \quad (6)$$

O'CONNEL has made a similar investigation. He has plotted the two periods against each other and found that the points satisfy a curve, which seems to approximate to a hyperbola.

⁸ Leningrad University Astronomical Observatory Bulletin No. 2 (1933).

⁹ HB 893 (1933).

¹⁰ MN 94.857 (1934).

J. VAN DER BILT has discussed the works of the previous two authors and repeated the investigation, adding five more variables to the list. The total number of variables with more than one period thus amounts to fourteen. His new list of periods permits not only the formation of the ratio p , but also of q (i. e. the ratio between the third and the second period). The first period P_1 and p when plotted against each other give a fairly straight line. Along this line, however, many values are missing. VAN DER BILT points out that this need not mean the non-existence of variables with these ratios and periods. The missing values are found for $p = 1$, where $P = 90^d$ to 100^d . It should be remembered that the difficulty of separating the two waves increases, when the two periods are approaching each other, that is when $p = 1$.

A curious fact is the constancy of the ratio q , which is distributed around the value of 6 with a very small dispersion.

When working with the light-curves available there was noticed, as has been mentioned above, an indication that ten small waves constitute one long period, on an average. During the course of this work P. P. PARENAGO¹¹ has independently made the same observation.

From a list of 18 stars, which PARENAGO has separated into AI Sco-stars (6 stars) and μ Cep-stars (12 stars), the following relations are obtained between the periods:

$$\log (\log P_2 - 2.75) = 2.4 \log P_1 - 4.95, \text{ [AI Sco-stars]} \quad (7 \text{ a})$$

$$\log P_2 = 0.58 + 1.20 \log P_1. \text{ } [\mu \text{ Cep-stars}] \quad (7 \text{ b})$$

In the case of the μ Cep-stars PARENAGO noted that $P_2 \simeq 10 P_1$.

The stars with two or three periods available in the present investigation have been brought together in Table 1. The values of the long periods must in many cases be regarded as especially uncertain, owing to the difficulties in deriving them. Several of them have been determined in a rather approximate way from the above-mentioned compilation of light-curves. The list contains 42 variables showing multiple periods. Further, 13 stars have been added, for which it has been possible to establish only the order of magnitude or the lower limit of the long period. There is also included a long-period variable, SV And (M7e, 7^m.8—14^m.0), which has been found to possess three periods.

From the list it will be seen that the ratio between the periods is not a constant. However, when ranging the p_1 's according to spectral classification an interesting result is obtained. For the K-stars we obtain two series of values: one of $\bar{p}_1 = 16.2$, another of $\bar{p}_1 = 28.9$ with three stars in each group. It is to be observed that the latter value is about twice the first. For the M-stars we find: $\bar{p}_1 = 11.4$ ($N = 24$). The N-stars, further, have a mean value of $p_1 = 7.7$

¹¹ Ver Sterne Bd V No. 5 (53) (1937).

TABLE 1.
Irregular variables with multiple periods.

Var.	<i>Sp.</i>	P_1	P_2	P_3	$\frac{P_2}{P_1} = p_1$	$\frac{P_3}{P_2} = p_2$
RV Cyg	N5	75 ^d	470 ^d		6.3	
RS Cam	Mb	88.7	960		10.8	
RS Cnc	M6	239	1460		6.2	
RT Cap	N3	211	1152		5.5	
VY Cas	M5	100	670		6.7	
ϱ Cas	cG5	200 } 365 }	1100		5.5	
T Cen	M0e	90.75	630		6.9	
SX Cen	F5	16.5	725		43.9	
RU Cep	K8	117.0	1600		13.7	
SS Cep	Mb	97.5	1435		14.7	
AF Cyg	M4	91	900		9.9	
DF Cyg	cK0v	49.8	782		15.7	
U Del	Mb	90 } 60 }	900		10.0	
RY Dra	N4p	125 } 330 }	1200		9.6	
TX Dra	M5	76.57	500		6.5	
UU Dra	M6	120	960		8.0	
Z Eri	Mb	74.3	1200		16.1	
SU Gem	K6v	24.9	716		28.7	
α Her	M5	89.2	2000		22.4	
V Hya	N6	527.4	6500		12.3	
X Mon	M3e	151.2	1400		9.3	
TT Oph	F8pe	61.08	2520		41.3	
W Ori	N5	200	2000		10.0	
α Ori	M2	11	250	2070	22.7	8.3
TW Peg	M8	90	934		10.4	
S Per	M5	840	1120	3360	1.33	3.0
TX Per	Mp	77	400		5.2	
UZ Per	M4	90	920	5000	10.2	5.4
ϱ Per	Mb	33.3	910?		27.3	
V Sge	Ob	17.17	530		30.9	
AI Sco	K	35.6	980		27.5	
Z UMa	M6e	198.0	2600		13.1	
RX UMa	—	89	610		6.9	
RZ UMa	M7	136.4	1300		9.5	
ST UMa	Mb	81	500		6.2	
V UMi	M4	70.5	1460		20.7	
S Vul	cK7v	67.77	1300		19.2	
RY UMa	M4e	41.42	311.1		7.5	
RV Tau	K0	39.25	1200	8000	30.6	6.7
μ Cep	M2	40- } 100 }	900	4500	22.5 } 9.0 }	5.0
VV Cep	M2ep	160	900		5.6	
UU Aur	N3	235	600	1970 } 3040 }	2.5	3.3 } 5.1 }
X Her	Mc	100.2	403	> 1000		
SV And	M7e	316	930	5400	2.94	5.8
RV Boo	M7e	120	700			
TV And	M4	114.8	490?			
U Cam	N5	300 } 418.7 }	> > 1400			
RY Cam	M3	135.4	> 1000			
RR CrB	Mb	56.8	> 700			

Var.	Sp.	P_1	P_2	P_3	$\frac{P_2}{P_1} = p_1$	$\frac{P_3}{P_2} = p_2$
S Dra	Mc	170 ^d				
		342 }	$> 1200^d$			
UX Dra	N0	140	$> 2 \times 5000$			
R Lyr	M5	45—75 }	1500			
		15—60 }				
W Per	M9	496? }	850			
		358? }				
SW Per	M4	83.6	> 1000			
V UMa	M3	207.5	$> > 1200$			
W Vul	M4e	238.7	> 1200			

($N=6$). The p_1 -values of these three spectral classes, therefore, form a series with decreasing p_1 for advancing spectral class.

We have:

$$\begin{array}{lll}
 \text{Spectral class } K: & p_1 = 16.2 [N=3]; (\text{or } 28.9) [N=3] \\
 \text{» » } M: & p_1 = 11.4 \pm 1.2 [N=24] \\
 \text{» » } N: & p_1 = 7.7 \pm 1.5 [N=6].
 \end{array}$$

The total mean value is found to be $p_1 = 12.6$.

The dependence of the p_1 -values on spectral class has been determined on the basis of 39 stars in the above list. The stars V Hya and S Per were excluded, since their periods are exceptionally long. There is no spectrum classified for RX UMa.

The following values of the characteristics and their mean errors were obtained:

$$\text{Mean value of spectral index: } \bar{s} = 6.01 \pm 0.16$$

$$\text{Mean value of ratio: } \bar{p}_1 = 14.3 \pm 1.6$$

$$\text{Dispersion in spectral index: } \sigma_s = 1.02 \pm 0.12$$

$$\text{Dispersion in ratio: } \sigma_{p_1} = 10.1 \pm 1.1$$

$$r_{s, p_1} = -0.818 \pm 0.053. \quad (8)$$

For p_2 a separation according to spectral class cannot be made, because of the small material available. Still, a mean value of the p_2 's has been formed and found equal to 5.3 ± 0.9 . As mentioned above, the same value found by VAN DER BILT is 6.

Since the p_1 -values are negatively correlated to spectrum, they may be expected to be correlated to the original period p_1 in the same way, because of the positive correlation between period and spectrum. In this calculation too, the stars V Hya and S Per with exceptionally large period have been excluded. There remained a list of 40 stars. The treatment of that list has resulted in the following data:

Mean value of period: $\bar{P}_1 = 99^d 0 \pm 9^d 0$

Mean value of ratio: $\bar{p}_1 = 14.3 \pm 1.6$

Dispersion in period: $\sigma_{P_1} = 56.8 \pm 6.3$

Dispersion in ratio: $\sigma_{p_1} = 10.1 \pm 1.1$

$$r_{P_1, p_1} = -0.689 \pm 0.083. \quad (9)$$

The values of the ratio p_1 decrease with increasing values of the original period of variation as well as with spectral index. The coefficients of correlation are undoubtedly real. However, in spite of the high values of the coefficients of correlation, these numbers may be misleading. The uncertainty in the p_1 -values must be remembered. It may be that the large extreme values are incorrect because of a missing of still another period for the short-period stars. Such an error would certainly influence the correlation to a considerable degree.

On the other hand an acceptance of the correlation as real might disclose interesting possibilities for our conceptions of the light-variation. The correlation shows that the redder the spectrum, the smaller the ratio between the long and the short period; and the larger the period, the smaller p_1 becomes. Or in other words, the short-period variables need more periods to produce one long wave than do the long-period ones. This might be a consequence of the interference of the original waves. It might also suggest that the long wave is not caused by interference alone. It might also be caused by some internal processes in the stars. Because of the positive correlation between period and spectrum referred to above, the effect would be present in both cases.

It is also of interest to observe that the mean value of p_1 for twelve stars with periods between 88^d and 120^d is 9.76. This is also near the mean value of p_1 for the M-stars. Further, this value of the ratio was observed from the light-curves in many cases.

It is, however, too early to allow any deductions to be made from the figures obtained. The dispersions in p_1 have been calculated for the M- and N-stars and the same for the p_2 -values. The values obtained are rather high.

$$\sigma_{p_1}(\text{M-stars}) = 5.93 \pm 0.86$$

$$\sigma_{p_1}(\text{N-stars}) = 3.67 \pm 1.06$$

$$\sigma(p_2) = 2.11 \pm 0.61.$$

The decreasing series of the p_1 -values may be real and may very likely imply a connexion with the physical condition of the star itself and hence the source of variability. However, the large dispersion makes any conclusions uncertain.

4. The RV Tauri variation.

A special case of variables with multiple periods is the class of RV Tau-stars. This class is defined by a light-curve showing a secondary maximum separated from the primary maximum by a secondary minimum, the latter

dividing the period into two halves. Variables of this class, which are few in number, have been further recognized by a rather small amplitude and spectra of classes K and M. A »regular» RV Tau-variable would have its main period divided in half by the secondary minimum.

On closer scrutiny the RV Tau-variables do not show very great regularity with regard to their light-variation. Frequently the secondary maximum is absent and in many cases the light-curves show a rather irregular appearance. JACCHIA has also found RV Tau-anomalies in the light-curves of many variables of the intermediate group, as well as among the long-period ones. The presence of anomalies of this kind is apparently very common among red variables. As examples may be mentioned the curves of S Dra, Z UMa, S Aql, RZ UMa, SW Per. But we shall find secondary maxima in almost every one of the intermediate variables. In no case, however, does the secondary maximum have the same position on the curve period after period.

The presence of the secondary maximum in the light-curves of the variables of the intermediate group might be explained by the interference of two or several periods discussed above. At certain phases the two maxima will be in such a position to each other that a »typical RV Tau-curve» is produced. Then the second maximum is obscured (R Sge, RZ UMa) and we observe a variation with single maxima. Later on the interference will cause the two periods to cancel each other, and the result will be a very slight irregular variation in light with a very small amplitude. An observation of the variable R Sge round the epochs JD 242 2200—2300 and RZ UMa round 4900—5200 would be represented by a very beautiful, regular light-curve of RV Tau-type, which, however, later degenerates.

It might therefore be supposed that the »RV Tau-class» really is a temporary special case of two interfering periods, the same as is generally characteristic of the variables of the intermediate group.

A permanent RV Tau-variable might be produced by certain values of the periods and amplitudes of the original variations.

It may be of interest to consider the individual stars classified as RV Tau-stars. To this end these stars have been collected by the present author from the ephemerides of two different years, thus showing a change possibly occurring in the description. The ephemerides of PRAGER for the year 1936 was used first. The number of RV Tau-variables here is 17, of which seven stars were not known with regard to their spectra. In this material eight are M-stars and three K-stars with periods ranging from 64^d to 342^d.

Later observations have, however, shown several of these stars to possess irregularities in their variation. Four of them have been referred by JACCHIA to the μ Cep-stars and AF Cyg-stars, five have the RV Tau-type (doubtful).

On revision of the list by means of the »Katalog und Ephemeriden veränderlicher Sterne» of 1938, nine of the stars have been found to possess irregularities. Six new cases have entered the list, the type of five of which is queried.

The real RV Tau-stars would be represented by the special value of $p = 2$ in the above-mentioned scheme of VAN DER BILT. It is well known that the type-variable of the RV Tau-class, RV Tau itself, shows a light-curve subject to a certain irregularity.

The complexity of the variation of RV Tau has been clearly proved by VAN DER BILT in his extensive investigations on the variation of this star in the year 1916.¹² RV Tau has always showed irregularities in its variation, and ENEBO (EINBU) proved the presence of a long-period variation in the maximum light, while minimum remained constant.

The period of this variation was found to be about three years, the short period being about 39^d.

We have here too, in fact, a typical star of the complexly varying class. But a variation of this kind was regarded as a unique occurrence among the variable stars.

VAN DER BILT finds the short period of 39^d to be the most regular feature in the variation of RV Tau. All other phenomena were found to be more or less irregular. The form of the curve is continually changing. At some epochs the extreme phases are sharp, at other epochs flat. The positions of maxima and minima differ from one phase to another.

The light-curve was investigated by the methods of harmonic analysis. In order to study the undisturbed short-period variation, he eliminated the long period variation, i. e. levelled the curve. The principal characteristic was then found to be a wave of 39^d25 in length. However, this wave was constantly disturbed by waves that were nearly of the same length. Further, the amplitude was found to change with a period of 1000^d. On the basis of his work VAN DER BILT classified RV Tau among the semi-regular variables.

Later¹³ the same author has found the following periods to satisfy the light-variation of RV Tau:

$$P_1 = 39^{\text{d}}25, P_2 = 1220^{\text{d}}, P_3 = 8000^{\text{d}}?$$

RV Tau has also been investigated by ZESSEWITSCH,¹⁴ who found that a short period of 78^d6 dominated the light-curve, in addition to a long-period wave of 1360^d.

The results related above tend towards the conclusion that there may be no difference between RV Tau-variables and other variables with complex light-curves. The RV Tau-phenomenon is simply a temporary consequence of this kind of variation. Most of the semi-regular variables with two interfering periods will at some time or other show RV Tau-anomalies.

The variable RY Cam offers a good illustration of the effect on the light-curve

¹² Rech Astr de l'Obs d'Utrecht VI (1916).

¹³ MN 94 846 (1934).

¹⁴ Leningrad Obs Bull No. 2 p. 2 (1933).

of the interference of waves. According to an investigation by BEYER¹⁵ the variation of this variable shows the following characteristics: besides the main variation with a mean period of 135^d.4 another periodicity of about 154^d is to be observed. From the interference of these two periods there will at times result a curve with strong resemblances to the RV Tau-curve.

The RV Tau variables were not supposed to show irregularities in their light-curves. Therefore RY Cam was not classified as an RV Tau variable.

DOBERCK characterizes the curve derived from observations in the years 1920—24 as apparently totally irregular.¹⁶

The results of an investigation on V Vul by GERASIMOVICH¹⁷ are also interesting. This star was found to have a variable light-curve, sometimes exhibiting the characteristic RV Tau appearance, sometimes resembling a Cepheid. The spectrum varied from cG7 at maximum to cK0 at minimum. Later on the average spectrum was given as cG5pv. The maximum magnitude was found to be fairly constant, while the minimum magnitude underwent long-period variations with an amplitude of 1^d.2. In addition, the period of the variable was suddenly subject to a change at the end of the year 1916. The change was observed in the maxima only. This »jump» increased the period and also the stability in the maximum phase, as the deduced average deviations in the two intervals show.

As is to be seen from the list of RV Tau-variables, this phenomenon is not limited to certain periods. It is observed all through the scale of periods.

A reference may be made to JACCHIA's light-curves of R Cen ($P=540^d.2$), R Nor ($P=488^d.3$), U CMi ($P=411^d$). These light-curves, however, represent two or more periods in each case. Therefore it is not out of the question that longer series of observation might disclose irregularities in their variation. Other light-curves of Me-variables, obtained by the same author — SV Her ($P=240^d$), R UMa ($P=301^d$), α Cet ($P=332^d$) — do not show any anomalies during the observed epochs. The variable V Cam ($P=514^d$), however, shows a less regular light-curve than the above mentioned stars. In this case there is an indication of a secondary wave on the ascending branch.

In the light-curves of the Cepheids there may be traced the last remains of a complex variation in the hump on the descending branch.

It might seem, from this, that the stars having red spectra favour the complex variation, while the stars of earlier spectral classes favour special values of the ratio p_1 and show few irregularities in the light-curves.

On the basis of the above considerations the RV Tau-stars have been included in the present investigation, since there seems to be no real difference between this group of variables and other members of the intermediate group.

¹⁵ Erg AN 8, C 27 (1904).

¹⁶ See also JACCHIA: Bologna Publ 2 p. 236 (1933).

¹⁷ HC 321 (1927).

5. Irregular variability in long-period variables.

The RV Tau-anomalies in the light-curves of the long-period variables have been especially considered above. The presence of this form of irregularity will, however, suggest the presence of irregularity in general in the light of these stars. The discussion of observations from later years has proved this to be true in several cases. Therefore the number of the irregular long-period variables will increase with increasing accuracy of observation. As examples may be mentioned SV And and RY UMa. The former star has been studied by J. VAN DER BILT,¹⁸ who deduced the following periods: $P_1 = 316^d$, $P_2 = 930^d$, $P_3 = 5400^d?$, and the latter by R. MÜLLER,¹⁹ who found the periods: $P_1 = 41.42$, $\bar{P}_1 = 300^d$. The light-curve of this variable, as obtained by MÜLLER and reproduced in his paper, is very clearly of the type *B*, according to the present classification. The similarity to the light-curves of μ Cep and α Ori and others is apparent.

To mention another case: U Cam, being a long-period variable of spectral class N5, has been found to possess two distinct overlapping periods: one of these has a length of 418^d.7, the other of 300^d. These two interfering periods give the light-curve an irregular appearance. Another variable of this kind is T Cnc, with a period of 483^d. The existence of an interfering period is here indicated by the increasing value of $(m_{\max} - m_{\min})$.

S Per is also an exceptional star, with an amplitude of 5^m.0. There can, however, be no doubt of the complex structure of the light-curve.

In the present list of complex variables more stars of this kind are to be found.

It is perhaps not very surprising to find complex light-variation in the types of variables which have been regarded as regular. The long period and the large amplitude of the Me- and Se-variables favour the smoothing out of irregularities in the formation of the mean curve. The irregularities might easily be attributed to errors of observation and consequently not be regarded as real, so much the more when the observer is not especially looking for them or expecting them. However, irregularities were not unknown even at earlier epochs in the history of variable stars. It will be remembered that ARGELANDER²⁰ already discovered irregularities in the period of Mira, but found these irregularities to possess a certain periodicity.

It will be of great interest to see if further observations will reveal variations of irregular nature more generally among the long-period variables.

6. Irregular variables with several periods not overlapping.

In the light-curves of another interesting group of irregular variables two or more periods, *not overlapping*, can be distinguished. These periods never appear

¹⁸ MN 94 p. 846 (1934).

¹⁹ ZfA Bd 11 p. 88 (1936).

²⁰ AN 26.369 (1848).

at the same time. At a certain epoch one period satisfies the observations. Then follows an interval of totally irregular variation, after which the second period sets in, and so on. This rather strange group has very few members. The one first discovered and discussed was UU Her, with two interchanging periods of 45^d4 and 72^d6. Further might be mentioned V Lyn with three periods: 54^d, 87^d and 163^d; Z Cnc with three periods: 80^d, 114^d and 166^d; KQ Cen with two periods: 136^d and 159^d.

CHAPTER 3

The spectra of the irregular variables

1. Distribution of the spectral classes.

The present material contains 360 variables with known spectra. If other classes of variable stars are characterized by their restriction to a few spectral classes, then it is remarkable for the irregular variables that all spectral classes are represented. The greatest number of stars, however, belongs to the classes K, M and N. From Table 2 it will be seen that 54 % of the irregular variables are classified as M-stars. The N-stars, with 18 % of the available material, K with 12 %, and G with 7 %, are next in order. The remaining spectral classes are represented by very few members, as is seen from Table 2.

TABLE 2.

Spectrum	Number of stars
O	2
B	2
A	5
F	6
G	24
K	42
M	196
N	66
R	5
S	5
Pec	7
Sum	360

2. Comparison of the spectra of the irregular and long-period variables and a short survey of knowledge about the spectra of the latter stars.

Since the spectral classes of the irregular variables as well as those of the long-period variables mainly belong to the classes M or N, a study also of the spectra of the latter variables is of great importance for our conception of the irregular variables. The main spectral difference between the two types of variables was previously considered to be found in the presence of emission-lines in

spectra of long-period variables. Such lines were for a long time not observed to appear in the spectra of the irregular variables. However, as will be shown below, recent investigations tend to approximate the two types of variable spectroscopically to each other.

The emission-lines in spectra of long-period variables do not remain stationary throughout the period. They appear some time after minimum and disappear before the next minimum. On account of the long period and the large amplitude, the opportunity for observations of the bright lines in spectra of this class will be considerable. The irregular variables, however, have a small amplitude and a comparatively short period. For these reasons the chance of discovering possibly existent bright lines will be smaller. Yet emission-lines have been observed in the spectra of many irregular variables. The present list contains 18 intermediate variables with spectra noted as Me. Besides these stars, which are formally classified as Me, there are some stars where bright lines have been observed at the maximum but have disappeared around the minimum. I will note these cases:

V Aql (irr),	Np:	H_{α} bright and varies in intensity (Shane).
μ Cep (irr),	M2:	At the bright maximum in 1930 H_{γ} and H_{δ} appeared as emission-lines, showing a displacement relative to the absorption-lines of 3.5 km/sec to the red. The displacement seemingly varies with the variation in brightness.
VV Cep. (irr),	Map: M2ep gM2ep	Absorption spectrum of class M. Bright lines H , Fe , frequently displaced relative to the absorption-lines.
T Cen (90 ^d 75),	Max: K5e: Min: M3	The bright H -lines show a character which disagrees with what is customary for Me-stars. Soon after minimum they become strong, and attain maximum of intensity before maximum phase, decrease rapidly in strength, and disappear a few days after maximum.
V Vul (75 ^d 98),	Max: cG7 Min: cK0	cG5pv
SX Her (102 ^d 8),	Max: K2 Min: Mb	gG7ev
		Bright lines appear shortly after minimum and disappear 30 ^d —40 ^d before next minimum.

GERASIMOVICH (HB 869) states:

»SX Her belongs to a small group of variables with spectra of class K that are closely related to the long-period variables. These stars are of unusual interest in the study of the connexion between Cepheids and long-period variables.»

R Set	Interval G5 to Ma (Miss CANNON).
	Max: cG0
	Min: M' (DEAN B. Mc LAUGHLIN).
S Vul (67 ^d 77), cK7v	
	Max: K
	Min: M2 (Miss CANNON)

From L. HUFNAGEL's paper in HB 866 I note the following items concerning S Vul:

1. Light-curve shows a cepheid-like variation.
2. Mean spectrum and period fit into the period-spectrum curve.
3. Photographic range (1^m.2) is larger than visual range (0^m.5).
4. Photographic light-curve does not show any conspicuous RV Tau-characteristics.

These results would classify S Vul as a Cepheid. But there is also some evidence against this classification: —

1. The cycles show considerable deviations from a mean value.
2. The radial velocities and light-curve do not show a simple cepheid-like correlation.

From the cases of irregular variables mentioned above it is seen that bright lines will occur in the spectra of these variable stars under circumstances corresponding to those of the long-period variables.

Recent very interesting investigations¹ may give rise to the question whether the irregular variation is much more widely extended than has hitherto been supposed. Among these investigations may be mentioned the list and discussion of parallaxes and absolute magnitudes of irregular variables of class M by P. P. PARENAGO.² These variables include many stars with a very small amplitude, the variation of which has been discovered from observations with the photo-electric cell by J. STEBBINS. At present only apparently rather bright objects can be observed with the photo-electric cell. The results obtained, however, show that with high probability there exists a great number of variables of this kind.

The phenomenon of the »irregular» variability, which was for a long time regarded as a singular phenomenon, may in fact be a very common occurrence, particularly among the red stars.

Since the spectra of the two groups of variables are nearly identical, it may not be out of place to give a short survey of our knowledge about the spectra of the Mira-variables. These belong to the classes Me, N, Se, and a few to R. In various papers MERRILL has published his observations and theoretical discussions

¹ The most important investigations on this subject are cited below.

² RAJ XI.1 (1934).

of the spectra of these stars. In this connexion there may also be mentioned the papers of A. H. JOY,³ L. CAMPBELL and Miss ANNIE J. CANNON,⁴ and G. SHAJN.⁵

The characteristic feature in the spectra of M-stars (with or without emission-lines) is the presence of absorption-bands of TiO. The absorption-spectrum for maximum phase of these stars is the same as for normal (non-variable) M-stars.⁶ Other absorption-bands are caused by ScO (*o* Cet) (BAXANDALL⁷) and MgH (JOY). The behaviour of bands and lines in the spectra has been studied in detail for a number of long-period variables by MERRILL and BURWELL in the above-mentioned paper. The authors have arrived at several very interesting results, of which a short review will be given here.

The absorption-lines and -bands vary in intensity with the phase of the visual curve. The observed variations may be caused by corresponding rise and fall in temperature in the photospheres and reversing layers of the stars. Observations have shown the presence of the TiO-absorption-bands originating from the lowest atomic level to be fainter at maximum phase in the Me-variables than at minimum. These variations seem to be correlated more closely with the magnitude than with the phase. The strength of the emission-lines, however, does not seem to be related to the temperature of the photosphere, a conclusion supported by the following five points⁸:

1. The intensity-ratio H_γ/H_δ plotted as a function of the light-phase, has a total lack of symmetry with respect to the light-curve.
2. $\lambda\lambda$ 4202, 4308, 4571 and other lines that are conspicuous during the declining phase are not observable at the corresponding magnitudes *before* maximum.
3. Normal and forbidden lines of ionized Fe are present near minimum when the absorption-spectrum indicates a relatively low temperature for the reversing layer.
4. Absorption-spectra practically identical with those of typical long-period variables, *but without bright lines*, are exhibited by numerous stars, many of which vary irregularly through small ranges. The strongest bright lines are found in spectra of variables with long periods and large amplitudes.
5. The displacement-curve of the emission-lines does not agree with that of the absorption-lines.

Point 4 indicates a connexion with the irregular variables. We have here, in fact, a good description of the spectra of these variables.

The band-absorption seems to influence the whole visible spectrum, thus also the continuous part of the spectrum.

³ ApJ 63.281 (1926).

⁴ HB 862 (1928).

⁵ ZfA 10.73 (1935).

⁶ THACKERAY, MERRILL: ApJ 53.185 (1921), 58.215 (1923), 65.23 (1927), 71.285 (1930).

⁷ PASP 41.168 (1929).

⁸ These points are extracts from the papers cited above.

A reason can be given why the molecules of TiO may exist at an abnormally high level in the atmospheres of these stars, *above* the reversing layer, where the absorption-line spectrum originates. It would then be reasonable to suppose the absorption-lines to be influenced by this stratum of TiO-molecules. If this layer has a sufficient optical thickness, it might not let through any actual photospheric radiation in the region of the bands, but instead emit the corresponding wave-lengths from its own layer. If this is the case, the original lines from a low level might either disappear or perhaps be replaced by fainter lines. The observations support this supposition. The atomic lines become fainter, while the bands increase in intensity after maximum.

The most peculiar feature in the spectra of the Mira-variables is the appearance of bright lines around maximum. In most cases there are present lines of the Balmer-series. But lines of Fe or Fe^+ can also appear.

Of these lines the *H*-lines are the most important. All lines of the Balmer-series to H_γ have been observed in the Me-stars. A remarkable feature about them is that their relative intensities differ widely from those observed in the laboratory. The changes are in close relation to the run of the light-variation. From present observations it seems probable that the abnormality in intensity depends on the presence and the degree of band-absorption. It also deserves mention that excessive deviations from laboratory-results have been noticed only in stars with TiO-bands. On the other hand there is an obstacle to this view: if the emission-lines were influenced by the TiO-bands, they would have to be produced at a lower level than these. This would not be true in the Sun, for example, where the *H*-lines are originated in the outermost layer, i. e. in the prominences etc. However, conditions in the red giants may be quite different from those of the Sun. The fact that the *H*-line is of normal intensity and lies outside the region of the bands is in favour of the hypothesis mentioned.

Besides the *H*-lines, MERRILL has found forbidden lines of ionized Fe to be present in some variables: χ Cyg, R Leo. These lines appear only near minimum, and simultaneously with emission-lines of Fe and Mg.

The behaviour of the TiO-bands and ZrO-bands in the spectra of class *M* and *S* stars has been studied by R. S. RICHARDSON⁹ on the basis of data obtained by F. LOWATER¹⁰ and A. CHRISTY.¹¹ Of the bands in question the ZrO-bands are more stable than the TiO-bands at higher temperature.

In class *S* both kinds of bands occur simultaneously. When the TiO-bands disappear or grow very faint at maximum magnitude, the ZrO-bands still remain. Several other facts also seem to indicate that the density of the *S*-stars is smaller than that of the *M*-stars.

Near maximum of *S*-stars and *M*-stars a great part of the molecules of the oxides is dissociated, owing to the increased temperature and the small pressure.

⁹ ApJ 78.354 (1933).

¹⁰ Proc Phys Soc London 44.51 (1932).

¹¹ ApJ 70.1 (1929).

Since both these attributes vary simultaneously in the atmospheres of the red variables, the band-spectra present a multitude of complexities.

RICHARDSON has derived the degree of dissociation for a given pressure and variable temperature, and vice versa. The TiO-molecules are rapidly dissociated when the variable approaches maximum, and disappear from the spectrum at 3000°K . At that temperature the ZrO-molecules are only dissociated to 60 %. The actual limits of temperature and pressure are difficult to ascertain, owing to the meagre collection of data.

In earlier investigations a number of M-stars without emission-lines will be found included among the Mira-variables. These stars possess on an average shorter period and smaller amplitude than do the »real» Mira-variables. This might be explained by the circumstance that these stars in fact belong to the irregular variables to which a period has been assigned. Then a close examination of the light-curve alone would reveal the existence of the complex light-variation, and the interfering periods. Many investigators — e. g. L. CAMPBELL and ANNIE CANNON, H. LUDENDORFF, W. GYLLENBERG — have, however, suspected that these variables are not identical with the Mira-variables, and have treated them separately in their investigations. With the results of later examinations in view it would seem probable, as has already been mentioned, that the »irregular» variables and the Mira-variables are spectroscopically of one and the same kind. It might therefore be possible that the emission-lines in the spectra of the irregular variable stars, classified as *M*, have only been overlooked. We should remember that the spectra may have been obtained at a phase of variation where no emission-lines were visible.

On the other hand, the point of view must not be disregarded that the non-existence of emission-lines may in fact be real and form a characteristic feature of this type of stars.

3. The temperature and radiometric magnitude of the irregular variables.

Comparison with the long-period variables.

PETTIT and NICHOLSON¹² have derived the temperatures on the basis of the radiometric data of the long-period and irregular variables. The general results from 19 members of class Me and 2 of Se, of which 11 stars have been observed under better conditions than the rest, give an average temperature at maximum of 2350 K and at minimum of 1830 K. The radiometric mean amplitude is $0^{\text{m}}.9$, corresponding to a mean visual amplitude of $5^{\text{m}}.3$. The mean absolute bolometric magnitude is found to be $-3^{\text{m}}.2$, which corresponds to that of a Cepheid having a period of 10^{d} .

It is of interest to observe that the radiometric magnitude is brightest at a

¹² ApJ 78.320 (1933) = Mt Wilson Contr 478 (1933).

phase-value of 0.1, or 30—40 days after the visual maximum. The real maximum of energy occurs at 0.14 phase, or 50 days, after visual maximum. Maximum of temperature is reached nearly at the time of maximum light. The TiO-bands are faintest at maximum light and not at maximum energy.

For many long-period variables a pulsation corresponding to the Cepheids is proved. For six stars the diameters were found to be 18 % larger at minimum than at maximum. The average diameter of the observed stars at maximum is $^{\circ}025$ and at minimum $^{\circ}029$.

The variation of the irregular variables may very likely have the same cause.

The measurements of the diameter of α Ori made by PEASE¹³ with the interferometer prove a pulsation of this star. The diameter was found to vary between the limits $^{\circ}047$ to $^{\circ}034$ from minimum to maximum of brightness. The radius of the star was then found on the average equal to $329 \odot$.

An investigation on R Set by DEAN B. MC LAUGHLIN¹⁴ shows this star to increase its radius from maximum to minimum from 110 to $144 \odot$. The temperature at maximum is 4500°K and decreases to 3240°K at minimum. The author assumes the light-variations in variable stars to be, with great probability, a temperature phenomenon.

The ranges in radiometric magnitude of the irregular variables investigated are small and give very slight evidence of variability. The variation in total energy of these stars therefore seems to be inconsiderable. The case is similar with the N-variables. These stars have been found to radiate more like a black body than do the M-stars. For V Cyg one of the hitherto lowest known temperatures (1500°K) has been measured.

4. Influence of band-absorption on the visual range of variation.

When the temperature is low and has the critical value for the appearance of absorption-bands, a minute change in the temperature is able to produce a considerable change in visual magnitude. These bands will produce a decrease in the visual magnitude, whereas their disappearance will give rise to a larger amplitude and thus to a brighter maximum magnitude than would be produced by the increase in temperature alone. JOY¹⁵ has estimated that the band-absorption in the spectrum of α Cet causes a visual amplitude of about two magnitudes.

In the case of the irregular variables, which are characterized by a mean amplitude of $1^{\text{m}}5$, the total visual light may be accounted for by increased band-absorption. It has already been mentioned that PETTIT and NICHOLSON found the radiometric ranges of these stars to be very small. This indicates a very small variation in energy. The irregular variables, being in a stage of spectral

¹³ Erg d. exakten Naturw 10 p. 91 (1931).

¹⁴ Publ Michigan Vol VII No. 2 (1938).

¹⁵ ApJ 63.281 (1926).

instability, might easily pass over the limit where bands appear or disappear, even if the impulse — a change of temperature — is very minute. Owing to their special normal state, a relatively large visual amplitude would be the consequence. In this connexion several authors have drawn attention to the importance of band-absorption. We may mention: CECILIA PAYNE and TEN BRUGGENCATE,¹⁶ PAYNE,¹⁷ PETTIT and NICHOLSON¹⁸ and JOY.¹⁹

5. The colour of the irregular variables.

TEN BRUGGENCATE and others have obtained interesting results about the variation in colour of the red variables during the cycle. The M-stars do not grow redder from maximum to minimum, in spite of the fact that their spectra advance towards the red. This circumstance may be explained so, that the TiO-bands cut away more of the red part of the spectrum, the more intense they grow. When, therefore, a variable, whether irregular or long-period, changes its spectrum from K5 (at maximum) to M3 (at minimum), the colour-index is practically unchanged. In the spectra of the N-stars, on the other hand, the absorption-bands of C and CN occur in the violet part of the spectrum, and this will be cut away with growing intensity. Here the red part of the spectrum becomes more dominant, and consequently the star will have a larger colour-index in minimum than in maximum.

¹⁶ HB 876 (1930).

¹⁷ HB 894 (1934).

¹⁸ ApJ 78.320 (1933) = Mt Wilson Contr 478 (1933).

¹⁹ ApJ 63.281 (1926).

CHAPTER 4

The apparent magnitudes

In the present investigations the apparent magnitudes of the irregular variables have all been referred to the photographic scale. This has been done for merely practical reasons, since most of the magnitudes are given as photographic ones, particularly those of stars with unknown spectra. The visual magnitudes have been reduced to photographic ones by using the colour-indices of KING.¹

It may be remembered that in fact an error is introduced into the photographic magnitudes by this procedure, since an average value of the colour-index has been used for maximum as well as minimum. In several cases, where it has been possible to follow the spectra of the variables over the whole period of variation, it has been established that the stars of spectral class *M* do not change their colour during the light-variation. Stars of spectral class *N*, however, have a larger colour-index at minimum. Hence the photographic amplitude is larger than the visual, as is easily seen.

Suppose:

$$c_{\min} > c_{\max}.$$

Put:

$$c_{\max} = c \text{ and } c_{\min} = c' = c + \Delta c. \quad (10)$$

Then we have:

$$m_{\text{ph}}(\max) = m_{\text{vis}}(\max) + c, \quad (11 \text{ a})$$

$$m_{\text{ph}}(\min) = m_{\text{vis}}(\min) + c + \Delta c. \quad (11 \text{ b})$$

Now:

$$a = m(\min) - m(\max), \quad (12)$$

or:

$$a_{\text{vis}} = m_{\text{vis}}(\min) - m_{\text{vis}}(\max),$$

and:

$$a_{\text{ph}} = m_{\text{ph}}(\min) - m_{\text{ph}}(\max).$$

Thus:

$$a_{\text{ph}} = m_{\text{vis}}(\min) - m_{\text{vis}}(\max) + \Delta c.$$

We obtain finally:

$$a_{\text{ph}} = a_{\text{vis}} + \Delta c. \quad (13)$$

¹ From a table in GRAFF: Grundriss der Astrophysik p. 510 (1928).

However, the error referred to above can only be eliminated, when the variation in brightness and spectrum during the period is accurately known. In the present computations no account has been taken of the errors in question.

The apparent magnitudes are not, in fact, very accurate characteristics. From the nature of the variation of the brightness in this class of variables, even the definition of the extreme phases presents difficulties. The maxima and the minima vary with a period of their own. From one period of variation to another we shall find a different value of the magnitude in either maximum or minimum. As a rule the brightest and faintest values obtained have been used respectively. This will appear more clearly from an examination of the material of light-curves. Since the apparent magnitudes are used as references and the basis of reductions in many computations, the errors in these quantities which are caused by an incorrect definition of the extreme phases cannot be avoided. But these errors might with approximation be considered accidental.

The mentioned error in the amplitude above should, however, be of systematical nature.

In a previous chapter the light-curves have been discussed. It has not been possible to prove which phase ought to be regarded as the normal one. For several reasons, the minimum magnitude of the long-period variables was by GYLLENBERG ² considered to be the normal phase of these stars. The main reason was the fact that the absolute minimum magnitude has the smallest dispersion. Therefore the characteristics ought to be better defined at minimum than at maximum. However, the long-period variables with their large amplitude would have more chance to develop marked differences from minimum phase to maximum, whereas such changes would necessarily be less pronounced in the irregular variables, where both the amplitude and the period P_1 are rather small. It is therefore only to be expected that the two phases for these stars should be more consistent than in the case of the long-period variables. Here e.g. the two curves of distribution would be expected to show the same peculiarities.

TABLE 3.
Dispersion in m from observed max and min.

Var.	P	$\sigma m_{\max}$	$\sigma m_{\min}$	$N_{\max}$	$N_{\min}$
o Cet	332 ^d	0 ^m 48	0 ^m 26	14	16
UU Her	45, 72	.17	.18	32	49
S Vul	68	.11	.15	43	31
SV UMa	77	.10	.18	33	33
Z UMa	198	.07	.42	38	34
SX Her	103	.16	.14	44	37
AF Cyg	182	.24	.20 *	87	74
RT Hya	256	.21	.46 *	51	41
R Pct	173	.25	.46 *	29	29

* Photographic magnitudes.

² Lund Medd Ser II No. 54 (1930).

By the use of available data the apparent magnitudes for a series of maxima and minima of one and the same star have been collected. From this material the dispersion in the two phases has been computed. From Table 3 it will be seen that for one star one phase has the lowest value of the mentioned dispersion, for another star the other phase.

It is seen from the table that the typical long-period variable α Ceti has the largest dispersion for maximum phase. The value is, indeed, twice as large at maximum as at minimum. The eight irregular variables, however, show in six cases a larger dispersion at minimum than at maximum, whereas the two remaining stars have a larger dispersion at maximum.

The dispersion in maximum- and minimum phases as computed from the individual stars ($N = 353$) shows the following result:

$$\begin{aligned}\sigma_{m_{\max}} &= 1.^m45 \pm 0.^m05, \\ \sigma_{m_{\min}} &= 1.66 \pm 0.06.\end{aligned}\tag{14}$$

All stars:

$$\begin{aligned}\sigma_{m_{\max}} &= 2.^m20 \pm 0.^m06, \\ \sigma_{m_{\min}} &= 2.19 \pm 0.06.\end{aligned}\tag{15}$$

The two values must be considered to be equal, when the mean errors are taken into account.

Consequently the method of determining the dispersions in the extreme phases will not be sufficient in establishing the normal state of these variables.

Differences in the two phases for the irregular variables have been discussed in Chapter 3. The spectrum has in many cases been found to change from minimum to maximum, and indicate a larger spectral index in the former phase than in the latter. Such changes are of course, more difficult to establish for these stars than for the long-period variables, on account of the small amplitude.

The frequency-distribution of the apparent magnitudes (Table 4, Figure 10) of the irregular variables shows a pronounced principal maximum at $10.^m0$ (phot) for maximum, and at $11.^m5$ for minimum. Besides the frequency-maximum mentioned there is to be noted in both curves a small frequency maximum at the brighter magnitudes, with modal values at the magnitudes: $m_{\max} = 4.5$ and $m_{\min} = 5.6$. Though it consists of few members this primary group is clearly marked in both curves. We shall call this small group frequency-maximum *A*.

For reasons explained below the material was divided at the galactic latitude $\pm 30^\circ$, and the distribution of the two groups of variables obtained in this way was studied (Figure 11). This procedure gave the following result. The stars of low galactic latitude gave rise to a still more pronounced primary frequency maximum, while the stars outside $\pm 30^\circ$ did not show any such phenomenon at all. Thus in the case of the irregular variables the first frequency-maximum is caused almost entirely by stars with $|b| < 30^\circ$. This group consists of 16 stars.

TABLE 4.

Frequency-curve of the apparent magnitudes.

m	M a x i m u m				M i n i m u m			
	n_1	$\bar{m}_1$	N	$\bar{m}$	n_1	$\bar{m}_1$	N	$\bar{m}$
1.0—1.9	1	1.8	1	1.8	—	—	—	—
2.0—2.9	—	—	—	—	1	2.9	1	2.9
3.0—3.9	1	3.1	1	3.1	1	3.7	1	3.7
4.0—4.9	6	4.5	6	4.5	2	4.6	2	4.6
5.0—5.9	6	5.3	6	5.3	6	5.6	6	5.6
6.0—6.9	13	6.6	13	6.6	5	6.4	5	6.4
7.0—7.9	19	7.5	19	7.5	5	7.5	5	7.5
8.0—8.9	73	8.5	75	8.5	20	8.5	20	8.5
9.0—9.9	109	9.6	125	9.5	50	9.5	50	9.5
10.0—10.9	101	10.4	132	10.4	93	10.5	106	10.5
11.0—11.9	26	11.4	78	11.4	97	11.4	115	11.4
12.0—12.9	6	12.3	55	12.4	47	12.4	88	12.5
13.0—13.9	3	13.5	42	13.4	23	13.3	80	13.4
14.0—14.9	—	—	23	14.4	10	14.2	50	14.4
15.0—15.9	—	—	8	15.2	2	15.5	30	15.4
16.0—16.9	—	—	—	—	1	16.3	24	16.3
All	364		584		363		583	

 n_1 = Stars with known spectra. N = All stars.

TABLE 4 a.

Frequency-curve of the *minimum* magnitudes.[Maximum A].

$m_{\min}$	N	N	All stars
	$b \leq \pm 30^\circ$	$b > \pm 30^\circ$	
2.5—2.9	1		1
3.0—3.4	—		—
3.5—3.9	1		1
4.0—4.4	—		—
4.5—4.9	1		1
5.0—5.4	1		1
5.5—5.9	5		5
6.0—6.4	3		3
6.5—6.9	2		2
7.0—7.4	1	2	3
7.5—7.9	1	1	2
8.0—8.4	3	5	8
8.5—8.9	7	5	12

TABLE 4 b.

Frequency-curve of the *minimum* magnitudes of the long-period variables of spectral-class Me.

[Maximum A].

$m_{\min}$	N_1 $b \leq \pm 30^\circ$	N_2 $b > \pm 30^\circ$	N all	$\bar{m}$
8.0—8.4	1	—	1	8.0
8.5—8.9	—	—	—	—
9.0—9.4	—	—	—	—
9.5—9.9	—	1	1	9.9
10.0—10.4	—	5	5	10.1
10.5—10.9	1	3	4	10.7
11.0—11.4	6	2	8	11.1
11.5—11.9	6	7	13	11.7
12.0—12.4	4	1	5	12.1
12.5—12.9	8	3	11	12.6
13.0—13.4	3	3	6	13.2
13.5—13.9	—	12	12	13.7

TABLE 5 a.

Distribution of the apparent *minimum* magnitudes of the N stars.

$m_{\min}$	$\bar{m}$	N
5.0—5.9	—	—
6.0—6.9	—	—
7.0—7.9	—	—
8.0—8.9	8.7	7
9.0—9.9	9.6	9
10.0—10.9	10.4	28
11.0—11.9	11.4	29
12.0—12.9	12.4	20
13.0—13.9	13.4	8
14.0—14.9	14.4	7
15.0—15.9	15.2	9
16.0—16.9	16.1	1
All		118

TABLE 5 b.

Distribution of the apparent *minimum* magnitudes of the S stars.

$m_{\min}$	$\bar{m}$	N	
		all	irr
10.0—10.4			
10.5—10.9	10.90	1	1
11.0—11.4	—	—	
11.5—11.9	11.70	5	3
12.0—12.4	12.40	1	1
12.5—12.9	—	—	
13.0—13.4	13.00	1	
13.5—13.9	13.50	1	
14.0—14.4	14.10	2	
14.5—14.9	14.70	2	
15.0—15.4	15.20	1	
15.5—15.9	15.76	6	
16.0—16.4	16.20	5	
16.5—16.9	16.80	3	
17.0—17.4	17.50	1	
All		29	5

It is of great interest to study the distributions of the apparent magnitudes of other classes of variable stars. To this end the long-period variables of spectral class Me, the Se-variables and N-variables were investigated.

The proper-motion material of long-period variables of class Me was kindly put at my disposal by Dr. GYLLENBERG. It has been revised according to newly obtained data, and a new frequency-curve of the apparent magnitudes of these stars has been established. These variables show a very clearly marked frequency-maximum at the brighter magnitudes in the curve of the apparent minimum magnitudes. As was pointed out by GYLLENBERG,³ the maximum magnitudes do not show this peculiarity.

The original material of long-period variables was divided by GYLLENBERG at galactic latitude $\pm 30^\circ$. In order to obtain a strict comparison of irregular and long-period variables in our case, the same limit of separation has been used. For the same reason, too, the investigation was carried out for the minimum magnitudes.

Group *A* was found to be present in both frequency-curves of long-period variables, contrary to those of the irregular variables.

The main maximum for the long-period variables was found to lie at $m_{\min} = 15.4$.

Maximum A: (from the curve).

	$m_{\min}$	
11 ^m .4	12 ^m .6	$ b < 30^\circ$
10.2	11.7	$ b > 30$
11.7		All stars.

The variables of spectral classes S and N have been investigated in the same way. The frequency-curve of the latter variables does not give any contribution to the present discussion, since the maximum at the brighter apparent magnitudes is completely missing. The stars of class S, though few in number, show a well pronounced bright-magnitude-maximum, the mode of which is at $m_{\min} = 11.7$ and nearly coincides with the mode of maximum *A* in the distribution of the Me-variables. The principal maximum is noted to be at $m_{\min} = 15.8$. The main maximum of the N-stars is at $m_{\min} = 11.0$.

The modal value of *A* of the irregular variables is at the minimum magnitude 5.6, and is thus about 5^m brighter than the corresponding maxima for the Me- and Se-variables.

It is of interest to observe that all three secondary maxima here considered are about 5 magnitudes brighter than the main maximum in the respective frequency-curves. Evidently the position of this maximum depends on the extension of the material. The possible importance of this fact will be further investigated below in connexion with discussion of the absolute magnitudes.

³ Lund Medd Ser II No. 54 (1930).

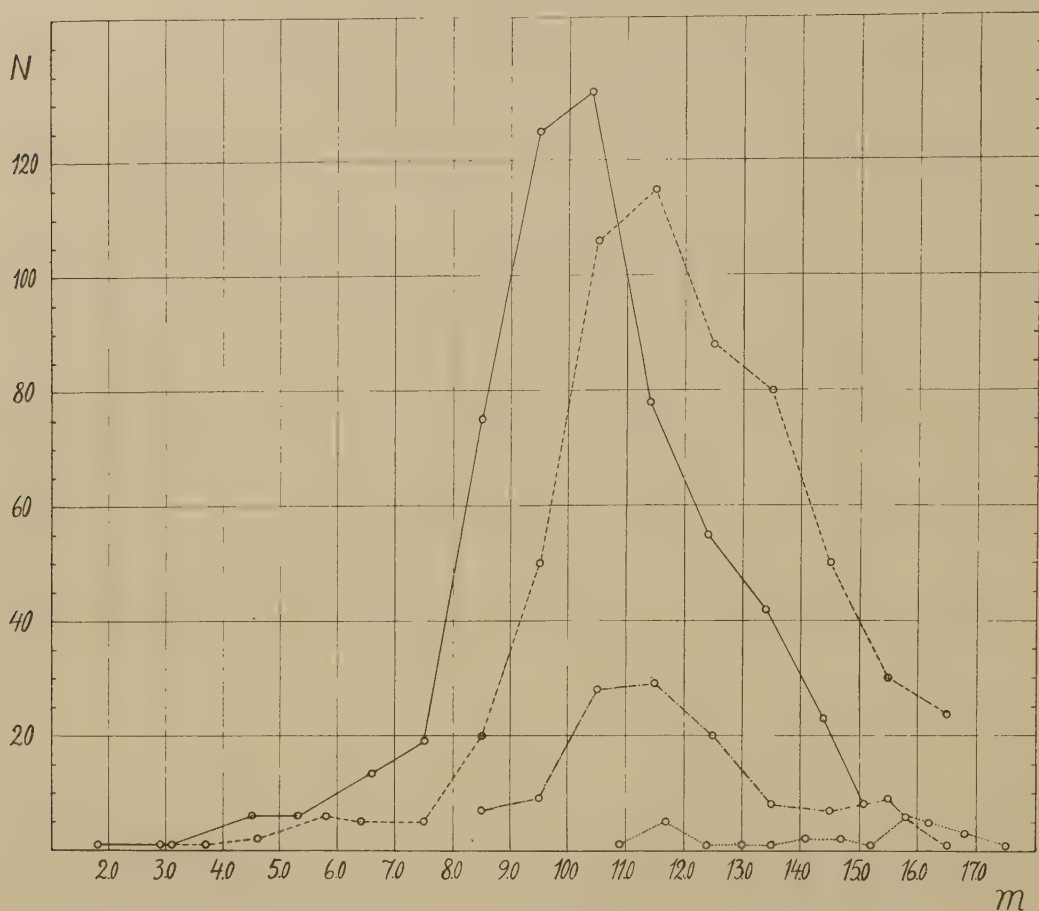


Fig. 10. Distribution of the apparent magnitudes of the irregular variables.

- maximum magnitudes of all irregular variables.
- minimum " " " " "
- - - minimum " " stars of spectral class N.
- minimum " " " " " S.

As has already been pointed out, there is a difference of about five magnitudes between the frequency-maxima in one and the same curve, and also between one maximum of the curve of the irregular variables and the corresponding maximum in the curves of other classes of stars.

The 19 or 16 stars belonging to the frequency maximum *A* of the irregular variables have been studied separately from different points of view, with the intention to obtain a possible explanation of the nature of this frequency-maximum.

Several possibilities present themselves. Since these stars are apparently bright, they may be quite near to us in space, thus forming a local star-cloud. The principal mass of stars may refer to clouds of stars at greater distances. If the

material at the faint magnitudes were reliable, we might thus be able to figure out several clouds or strata in the Galaxy.

If this explanation were the correct one, the 16 stars would be expected to be scattered over all galactic latitudes. This does not agree with the observed facts (See Table 29). The apparent galactic latitude is smaller than 30° . If the z -values are derived, the proximity to the galactic plane is still more evident.

Another possibility for these stars would be that they are absolutely very bright or super-giants. Then they would be expected to be found near the galactic plane, as also is the case.

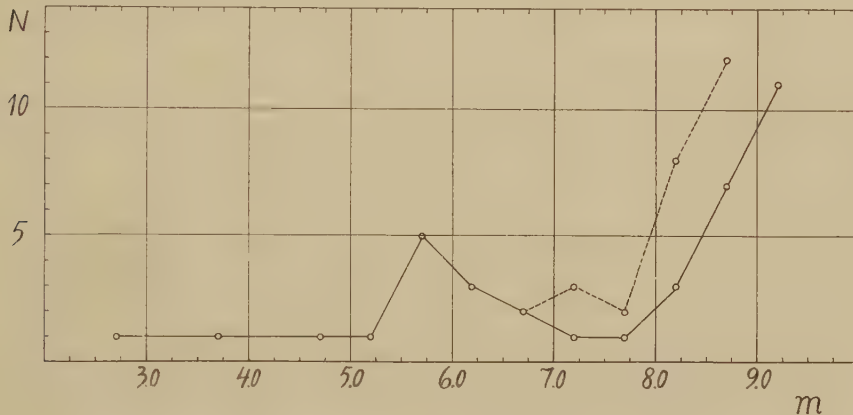


Fig. 11. Distribution of the *minimum* magnitudes of the irregular variables. [Frequency-maximum A .]

—○— stars of galactic latitude $\leq \pm 30^\circ$.
 - - -○- - all stars.

Of course there remains a third possibility, which must not be overlooked: viz. that of a combination of the two possibilities mentioned. The maximum A might hence consist both of nearby stars, apparently but not necessarily absolutely very bright, and of super-giants, lying in a galactic zone.

From the list (Table 29) it is seen that for 14 of the stars the spectrographic absolute magnitudes have been determined. These agree with the assumption that these stars are super-giants.

In all cases the galactic latitudes have rather small values. Therefore the probability that these stars are super-giants should be considered. It is observed that when the galactic latitudes are reduced by the photometric factor $10^{0.2m}$, the proximity to the galactic plane is still more evident. Therefore these stars must be considered with very high probability to be giants. The three variables in the group at high galactic latitude and fainter magnitude show a distance from the galactic plane that is far greater than that of the remainder.

CHAPTER 5

Statistical investigations of the characteristics of the irregular variables

The studies of the different classes of variables have in certain cases proved the existence of mathematical relations between the characteristics of the variation of these stars. The relations mentioned have, as is known, been of very great importance in the study of the variable stars. By means of these relations it is possible to find certain qualities of a variable, when another quality is known. It is, however, not necessary to enter here upon the well-known relations referred to.

The problem will be somewhat more difficult in the case of the irregular variables, owing to the nature of these stars. While the long-period variables and Cepheids generally have well defined periods and amplitudes, these characteristics are much less definite in the case of the irregular variables. Yet it has been thought worth while to try and investigate the relations between them.

The characteristics considered are: the periods P , the amplitudes A , the spectral index s , the apparent maximum and minimum magnitude m , the apparent and reduced galactic position $\sin b$ and $10^{0.2m} \sin b$.

The relations between these quantities have been studied both graphically and by means of the coefficients of correlation.

The relation between the logarithm of the period and the amplitude.

It is a well-known fact that there exists a real correlation between the logarithm of the period and the amplitude for the long-period variables¹ and for the Cepheids.² In both cases the longer period corresponds to a larger amplitude.

The irregular variables have been investigated below as to the relations between the above mentioned quantities.

Figure 12 a gives the mean values of the amplitudes computed for a given interval in $\log P$. The line of regression indicates a small increase in amplitude with increasing $\log P$.

¹ L. CAMPBELL, ANNIE J. CANNON: HB 862 (1928).

² K. LUNDMARK: Lund Medd Ser I No. 128 (1931).

The material with known periods, embracing 224 stars ($P \leq 180^d$), has been used for a determination of the coefficient of correlation. Using the individual values of P and A we have found:

$$r_{\log P, A} = +0.011 \pm 0.067. \quad (16)$$

The diagram indicates a difference in the stars represented by the first point of the curve from the other stars. Leaving these seven stars out of account, we obtain a somewhat higher value of $r_{\log P, A}$:

$$r_{\log P, A} = +0.121 \pm 0.066. \quad (17)$$

TABLE 6.
Period — amplitude — spectrum — relation.
[Abscissa: logarithm of period.]

P	$\bar{P}$	$\overline{\log P}$	$\bar{A}$	$\bar{s}$	$\bar{m}_{\max}$	$\bar{m}_{\min}$	$ \sin b $	$\left[\begin{smallmatrix} 10^{0.2m} \times \\ \sin b \\ (\text{min}) \end{smallmatrix} \right]$	N	
									With spectr.	All
$< 40^d$	28.9	1.442	1.95	4.24	9.21	11.15	.219	55.7	7	8
40—49	45.8	1.660	1.10	4.70	9.26	10.36	.343	66.6	4	5
50—59	53.5	1.727	1.50	5.77	11.03	12.53	.411	133.2	6	11
60—69	64.3	1.808	1.34	5.02	12.09	13.43	.240	145.2	6	19
70—79	75.1	1.874	1.43	5.44	10.47	11.94	.410	105.9	14	23
80—89	85.3	1.931	1.57	5.73	10.72	12.29	.375	155.3	15	29
90—99	93.1	1.969	1.77	6.17	10.06	11.97	.306	91.7	15	26
100—109	104.1	2.017	1.39	6.11	10.59	11.93	.357	120.3	12	20
110—119	114.1	2.057	1.86	6.10	11.64	13.49	.223	155.2	7	17
120—129	122.8	2.090	1.38	6.58	10.94	12.25	.257	77.9	10	16
130—139	134.3	2.128	1.51	6.32	10.75	12.26	.284	85.7	10	17
140—149	144.7	2.160	1.85	6.31	10.62	12.47	.254	104.8	9	13
150—159	154.0	2.188	1.78	6.32	11.06	12.84	.465	250.0	4	10
160—169	163.5	2.213	1.79	6.74	9.69	11.46	.398	157.9	5	7
170—179	172.9	2.237	2.07	6.37	10.07	12.13	.567	139.0	3	3
Mean:	99.43	1.965	1.585	5.89	10.71	12.30			127	224

Considering the mean error above, the value of $r_{\log P, A}$ can possibly be regarded as real.

The relative constancy of the amplitude is also proved by its small dispersion. For all stars ($N=580$) we obtain:

$$\bar{A} = 1^m501 \pm 0^m031, \quad (18)$$

$$\sigma(A) = 0.752 \pm 0.022. \quad (19)$$

The material with known periods ($N=224$) gives the following values:

$$\bar{A} = 1^m585 \pm 0^m046, \quad (20)$$

$$\sigma(A) = 0.689 \pm 0.033. \quad (21)$$

This statistical constancy of A may be regarded as an important part of the definition of these variables. In no other class of variables can the amplitudes be regarded as constant.

The line of regression, drawn in Fig. 12 a, is given by the following equation, using $r_{\log P, A} = +0.121$.

$$A = +0.5075 \log P + 0.588. \quad (22)$$

The second line of regression can be computed from the material. The equation of this line is:

$$\log P = +0.0289 A + 1.919. \quad (23)$$

The relation between the logarithm of the period and the spectrum.

A general period-spectrum relation for all classes of variable stars was first established by SHAPLEY.³ This relation has been used by several astronomers to classify the variables. Variables of the irregular type were found to lie outside the line, a fact which is of importance in GERASIMOVICH's classification of the intermediate stars. Variables which satisfied the curve were considered to belong to the »principal sequence» of variable stars.

However, the irregular variables, which did not satisfy the relation, are now known to embrace the whole sequence of spectra. Therefore, since they also are included in the whole series of periods, an independent period-spectrum relation may exist for these stars.

From the diagram in Fig. 12 b a positive correlation between the quantities $\log P$ and s is clearly to be observed, as in the general case of variable stars. The longer the period, the redder the spectrum.

In the computations the spectral indices were used, each spectral class being divided into ten sub-groups.

The mean value of the spectrum in the present material was found to be $K9 \pm 5$ subclasses.

The coefficient of correlation, computed for the material of 127 stars, is:

$$r_{\log P, s} = +0.739 \pm 0.040, \quad (24)$$

which means a pronounced correlation between these two parameters, in the same way as has been found for other classes of variables.

The curve represents one of the two lines of regression, written as follows:

$$s = +4.389 \log P - 2.729. \quad (25)$$

The other line of regression is:

$$\log P = +0.124 s + 1.232, \quad (26)$$

and is represented by the curve in Fig. 13 a.

³ HB 861 (1928).

A graphical relation between the logarithm of the period and the apparent maximum and minimum magnitudes and the galactic position.

The diagrams in Fig. 12 c representing the correlation between $\log P$ and apparent maximum and minimum magnitudes show a rather complicated appearance. The apparently brightest magnitudes are found for the shortest periods. Minimum of brightness occurs at about 60^d and 110^d , whereas stars with periods around 95^d are brighter than those with periods larger than 50^d . Clearly there is no linear correlation here. The minimum magnitudes give a curve corresponding in detail to that of the maximum magnitudes, except that the brightness of the magnitudes at the period-value of 95^d is less pronounced.

Since the galactic z -coordinates for these stars are of special interest, an investigation has been carried out to ascertain whether one or several characteristics might depend on the galactic arrangement of the stars. Such a characteristic is, of course, the masses of the stars, which can at present be determined individually only for very few irregular variables.

In Fig. 12 d the absolute values of $\sin b$ have been plotted against the values of $\log P$. In Fig. 12 e the absolute values of $10^{0.2m} \sin b$, reduced by means of the minimum magnitudes, represent the ordinates.

In the distribution in Fig. 12 d the longest periods are the most scattered. This phenomenon remains also in the reduced coordinates. Considering the distribution in Figs. 12 d and e, a difference between the stars of periods about 140^d and stars of other periods might be indicated. Perhaps they are more closely related to the Mira-variables than those with shorter periods.

The stars with periods shorter than 60^d are most closely concentrated to the galactic plane. This suggests a likeness to the Cepheids, which are of all variables known to have the greatest apparent galactic concentration.

The remainder of the stars show a rather small scattering around a mean value of the reduced z -coordinates.

A mere calculation of the coefficient of correlation would, as is evident from the diagrams, be misleading in the above related cases, and the graphical representation was therefore considered to be the best method of studying the relations above.

The relation between spectrum and amplitude.

The material for studying the relation between spectrum and amplitude embraces 353 stars. Table 7 a and b give the mean values, which are the basis of the following figures: —

For this material the mean value of the spectral index is found equal to:

$$\bar{s} = 6.11 \pm 0.05 \quad (\bar{s} = M 1.1 \pm 0.5). \quad (27)$$

TABLE 7 a.
Relation between spectrum and logarithm of period.

Spectrum		N	$\bar{s}$	$\overline{\log P}$
O	0	1	0.0	1.235
B	1	1	1.0	1.653
A	2	1	2.0	1.792
F	3	4	3.52	1.600
G	4	15	4.45	1.952
K	5.0—5.4	6	5.05	1.803
	5.5—5.9	7	5.61	1.915
M	6.0	11	6.00	1.999
	6.1	3	6.10	1.859
	6.2	7	6.20	2.045
R	6.3	24	6.30	1.973
S	6.4	12	6.40	1.939
	6.5	9	6.50	2.023
	6.6	7	6.60	2.083
	6.7	4	6.70	2.028
	6.8	5	6.80	2.012
	7.0—7.4	10	7.25	2.120
	7.5—7.9	—	—	—
		127		

TABLE 7 b.
The spectrum, correlated to the amplitude, the apparent magnitude and the galactic position.

[Abscissa: spectrum.]

Spectrum		N	$\bar{s}$	$\bar{A}$	$\bar{m}_{\max}$	$\bar{m}_{\min}$	$ \overline{\sin b} $	$\left \frac{100.2m \times}{\sin b} \right $ (max)	$\left \frac{10^{0.2m} \times}{\sin b} \right $ (min)
O	0	2	0.00	2 ^m 30	10 ^m 60	12 ^m 90	.199	34.0	75.5
B	1	2	1.40	0.60	6.60	7.20	.017	0.5	0.5
A	2	5	2.14	1.20	8.22	9.42	.132	10.0	19.0
F	3	6	3.52	1.53	8.98	10.58	.531	31.7	63.7
G	4	24	4.39	1.73	9.44	11.20	.302	31.5	90.8
K	5.0—5.4	26	5.06	1.46	8.96	10.41	.356	25.8	54.9
	5.5—5.9	16	5.56	1.35	9.40	10.75	.183	16.1	30.2
M	6.0	30	6.00	1.40	10.00	11.39	.259	29.0	57.7
	6.1—6.2	18	6.17	1.24	9.08	10.32	.219	24.7	53.3
	6.3	59	6.30	1.33	9.26	10.59	.419	35.5	77.1
R	6.4	21	6.40	1.71	9.50	11.17	.404	36.0	79.0
S	6.5	21	6.50	1.77	9.22	11.00	.382	31.8	95.7
	6.6—6.7	22	6.63	1.55	9.62	11.13	.460	40.5	86.2
	6.8—6.9	35	6.80	1.41	9.23	10.65	.456	33.7	64.3
	7.0—7.2	15	7.03	1.35	9.57	10.92	.218	18.1	29.3
	7.3	44	7.30	1.41	9.65	11.05	.211	18.1	36.2
N	7.4—7.9	7	7.53	2.11	9.10	11.21	.307	15.7	90.4
Mean:		353	6.115	1 ^m 470	9 ^m 374	10 ^m 840	.3327	28.54	63.59

The mean value of the amplitude is:

$$\bar{A} = 1^{\text{m}}.47 \pm 0^{\text{m}}.04 \quad (28)$$

$$\sigma(A) = 0.783 \pm 0.029. \quad (29)$$

From the graph (Fig. 13 b) it is evident that there is no dependence of the amplitude on spectrum, which is also confirmed by the value of the coefficient of correlation, computed from the individual stars, which is found to be:

$$r_{s, A} = -0.017 \pm 0.053. \quad (30)$$

Though the correlations between the amplitude and the logarithm of period and the spectrum are very small, it might still be of interest to compute the coefficients of the triple correlation between these three parameters, and derive the lines of regression.

We have found:

$$r_{\log P, A} = +0.121$$

$$r_{\log P, s} = +0.739$$

$$r_{s, A} = -0.017.$$

Then the multiple coefficients of correlation R_{ij} have the following values:

$$R_{\log P, A} = +0.198$$

$$R_{\log P, s} = +0.747 \quad (31)$$

$$R_{s, A} = -0.159.$$

We have the following lines of regression:

$$\log P (A, s) = 0.0291 A + 0.1239 s \quad (32 \text{ a})$$

$$A (\log P, s) = -0.1797 s + 1.1351 \log P \quad (32 \text{ b})$$

$$s (\log P, A) = 4.499 \log P - 0.1409 A. \quad (32 \text{ c})$$

A study of the multiple correlation might be of still greater interest as regards $\log P$, A and s in the case of e. g. the long-period variables, where there appears a real correlation between these three parameters.

The relation between spectrum and apparent maximum and minimum magnitude.

A study of the graphical representation of the relation between spectrum and the apparent magnitude for 353 stars (Fig. 13 c) suggests a slight dependence,

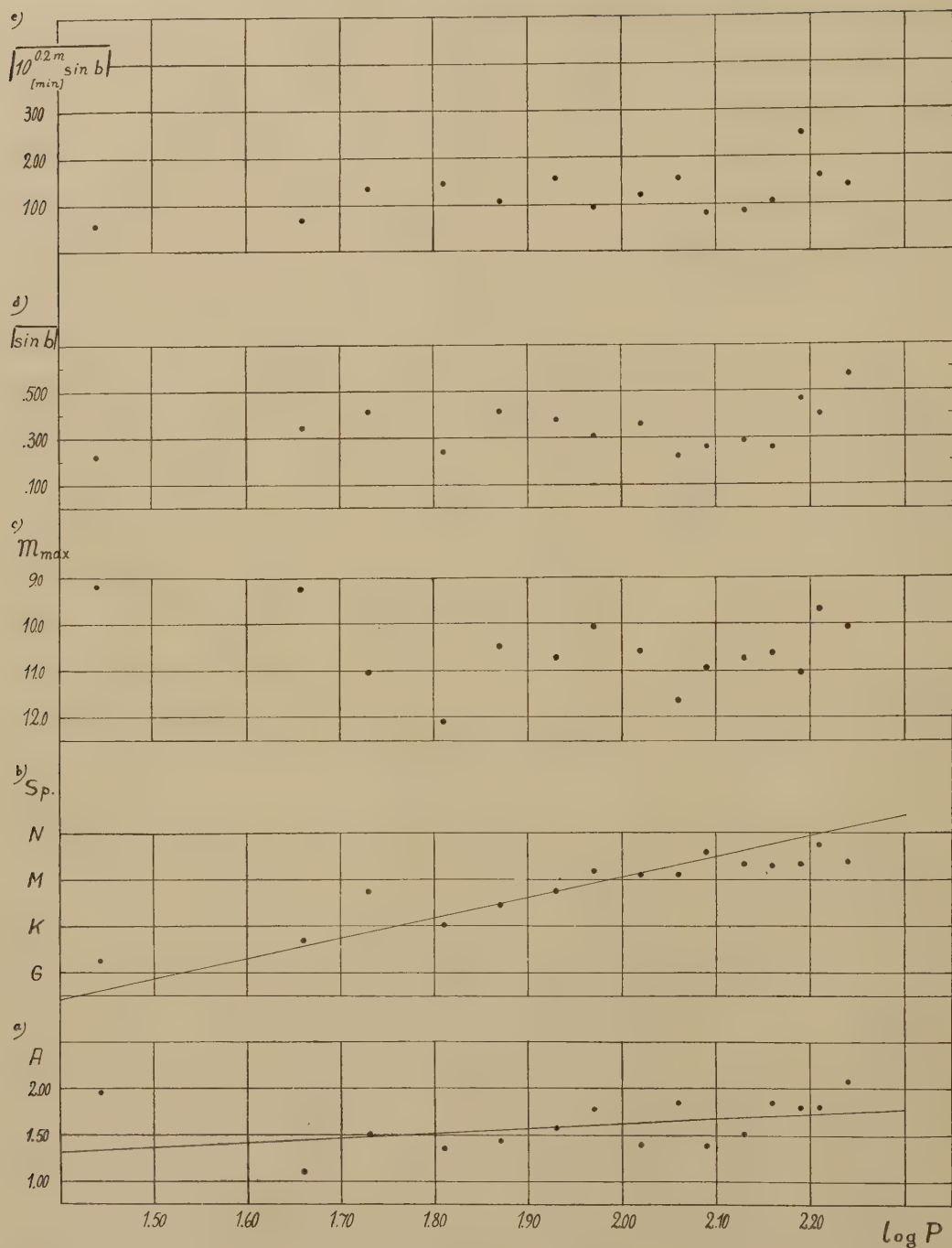


Fig. 12. The correlation between the logarithm of the periods of the irregular variables and the amplitude, the spectrum, the apparent magnitude and the galactic z' -coordinate. The figure gives the mean values of the characteristics for a given logarithm of period.

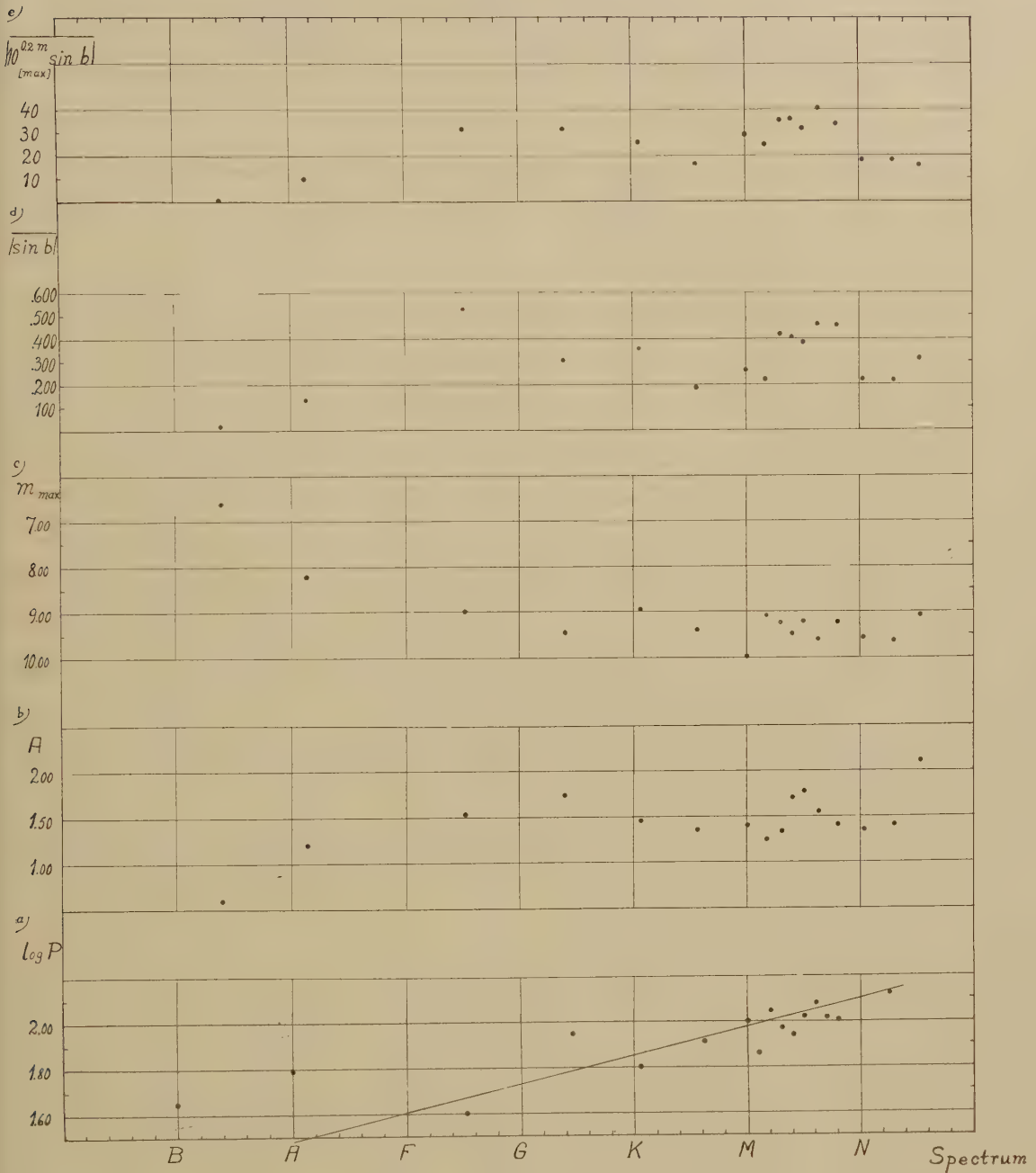


Fig. 13. The correlation between the spectra of the irregular variables and the logarithm of the period, the amplitude, the apparent magnitude and the galactic z' -coordinates. The figure gives the mean values of the different characteristics for given spectrum.

at least for the maximum magnitudes. The minimum magnitudes lie somewhat more scattered than the maximum magnitudes in the diagram. The dispersion would hence be expected to be greater for minimum magnitudes than for maximum

TABLE 7 c.

Sp.	N	$\sin b$	$10^{0.2m}_{(\max)} \sin b$	$10^{0.2m}_{(\min)} \sin b$
G	24	.302	31.5	90.8
K	42	.290	22.1	45.5
M	206	.384	33.5	73.1
N	66	.223	17.8	40.4

magnitudes. A direct calculation of the dispersion from the present material gives as result the following values of this quantity for the two phases:

$$\begin{aligned}\sigma_{m_{\max}} &= 1^{\text{m}}.45 \pm 0^{\text{m}}.05 \\ \sigma_{m_{\min}} &= 1.66 \pm 0.06.\end{aligned}\tag{33}$$

The difference in the two dispersions seems, however, hardly to be real. The two extreme phases must be regarded as having the same dispersion. This fact distinguishes the irregular variables from the Mira-variables, which have been proved to have a definitely larger dispersion in their maximum magnitudes than in their minimum magnitudes.

When the whole of the present material is taken into account, the calculated dispersions in the maximum and minimum magnitude show the same value. We find for 580 stars:

$$\begin{aligned}\sigma_{m_{\max}} &= 2^{\text{m}}.20 \pm 0^{\text{m}}.06 \\ \sigma_{m_{\min}} &= 2.19 \pm 0.06.\end{aligned}\tag{34}$$

The coefficients of correlation are:

$$\begin{aligned}r_{s, m_{\max}} &= +0.104 \pm 0.053 \\ r_{s, m_{\min}} &= +0.064 \pm 0.053.\end{aligned}\tag{35}$$

Neither of these values is likely to be real. There seems to be no probability for the existence of a correlation between apparent magnitude and spectrum.

TABLE 8. The apparent magnitude correlated to the logarithm of period, the spectrum and the amplitude. [Abscissa: m].

$m_{\max}$	N_1	$\bar{m}$	$\log P$	N_2	$\bar{m}$	$\bar{s}$	N_3	$\bar{m}$	$\bar{A}$
$< 4^m0$	—	—	—	2	2^m45	5.50	2	2^m45	0^m80
4.0—4.4	1	4^m10	1.531	2	4.10	5.65	2	4.10	0.70
4.5—4.9	1	4.80	1.950	3	4.87	4.90	3	4.87	0.83
5.0—5.4	2	5.00	1.587	4	5.12	4.32	4	5.12	0.90
5.5—5.9	1	5.70	2.000	2	5.70	6.35	2	5.70	0.65
6.0—6.4	2	6.10	2.182	4	6.12	6.17	4	6.12	2.25
6.5—6.9	4	6.78	2.060	9	6.76	5.94	9	6.76	1.54
7.0—7.4	1	7.59	2.021	8	7.17	5.97	8	7.17	1.37
7.5—7.9	6			11	7.70	6.01	11	7.70	1.20
8.0—8.4	10	8.23	1.933	27	8.23	6.17	30	8.23	1.57
8.5—8.9	11	8.70	1.864	43	8.68	6.23	44	8.68	1.53
9.0—9.4	18	9.23	1.956	49	9.20	6.02	58	9.20	1.60
9.5—9.9	20	9.68	2.005	55	9.69	6.27	64	9.69	1.45
10.0—10.4	28	10.22	1.986	54	10.20	6.30	72	10.21	1.47
10.5—10.9	24	10.72	1.992	45	10.69	6.14	59	10.70	1.50
11.0—11.4	18	11.21	1.971	17	11.20	6.49	43	11.18	1.73
11.5—11.9	12	11.77	1.901	8	11.72	5.65	34	11.70	1.53
12.0—12.4	19	12.18	1.999	6	12.12	4.80	33	12.17	1.56
12.5—12.9	9	12.63	1.942	1	12.90	6.00	25	12.64	1.30
13.0—13.4	13	13.09	1.932	1	13.10	4.50	19	13.08	1.59
13.5—13.9	10	13.67	1.979	2	13.75	6.55	23	13.67	1.32
14.0—14.4	5	14.14	2.010	—	—	—	13	14.18	1.37
14.5—14.9	5	14.68	1.949	—	—	—	10	14.66	1.34
15.0—15.4	4	15.10	2.027	—	—	—	8	15.17	1.20
Mean:	224	10^m713	1.9666	353			580		1^m489

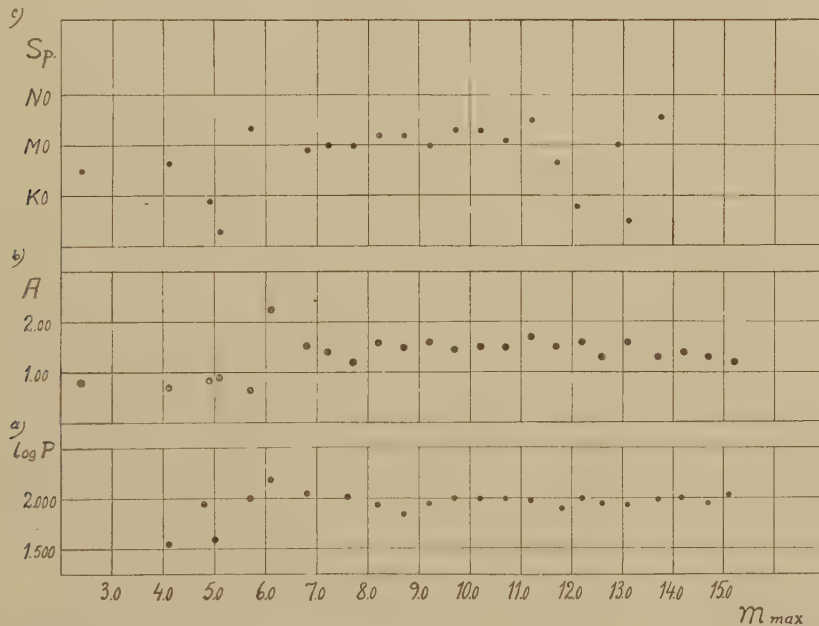


Fig. 14. The correlation between the apparent maximum magnitude and the logarithm of the period, the amplitude and the spectrum. The figure gives the mean values of the different characteristics for given apparent magnitude.

The relation of galactic position to spectrum.

The curves representing the correlation between the two quantities $\sin b$ and $10^{0.2m} \sin b$ and spectrum (Fig. 13 d and e) show a similar appearance to each other. The stars of spectral-classes B and A lie near to the galactic plane, both apparently, and when reduced by the photometric factor. The M-stars are most scattered over the sky, and the N-stars are nearer than the former to the galactic plane, thereby indicating that the N-stars have larger masses than the M-stars.

Table 7 c gives a concentrated view of the mean values of the z 's for each spectral class.

The regression-lines giving the logarithm of the period, the amplitude and the spectrum, when the apparent magnitude is known, are apparent from Table 8 and Fig. 14.

CHAPTER 6

The galactic distribution

I. The distribution in latitude and longitude

The irregular variables are distributed over all galactic latitudes, as is seen from Fig. 15 and Table 9. There is nearly the same number of stars observed on both hemispheres of the sky. Actually the number is somewhat larger for southern than for northern galactic latitudes.

The Cepheids, with their strong concentration towards the galactic plane, show a remarkable tendency to cluster in certain regions, some of which are found also to include irregular variables, e. g. Cygnus, Cepheus, Perseus, Orion, Scutum. The very pronounced concentration of Cepheids in Carina is, however, only to a small extent reflected in the distribution of the irregular variables. A tendency to cluster in the same regions may be used as a means of deriving the absolute magnitude of the irregular variables, since it is possible to determine the absolute magnitude for each Cepheid when its period is known. This matter will be discussed in another chapter.

The frequency-distribution of the irregular variables with regard to galactic longitude reflects a knotty appearance in Fig. 15 (See Fig. 16). We find here again as maximi-regions the clusters in Cygnus, Orion, and elsewhere.

If we disregard the high frequency at $l=165^\circ$ (Orion), the number of stars decreases on an average from $l=20^\circ$ to $l=180^\circ$, with a minimum of stars between longitudes 180° and 250° . The number of stars is 376 in the first half-part of the sky, against 206 in the second half-part.

The distribution-curve of the galactic latitudes shows the largest frequency between $b = \pm 25^\circ$. The distribution is a skew one. The frequency has its mode-value at $b = -7.5^\circ$, as read from the curve, while the mean value is found at $b = +2.3 \pm 0.97^\circ$.

The distribution of the coordinates has been computed for the irregular variables. The computation has been restricted to embrace the parameters of the second order, i. e. those which determine an ellipsoidal distribution. Stars with $m_{ph}(\text{max}) \geq 11.0$

TABLE 9.
Distribution in galactic latitude and longitude.

b	N	$\bar{b}$	l	N	$\bar{l}$
$+ 85^\circ + 89^\circ$			$0^\circ - 9^\circ$	19	5.1
80 84	1	$+ 84.$	10 — 19	23	15.3
75 79	1	$+ 75.$	20 — 29	30	25.5
70 74	4	$+ 72.5$	30 — 39	20	33.2
65 69	7	$+ 66.3$	40 — 49	36	43.3
60 64	6	$+ 62.8$	50 — 59	22	54.0
55 59	8	$+ 57.2$	60 — 69	26	65.9
50 54	9	$+ 51.3$	70 — 79	20	73.5
45 49	8	$+ 46.6$	80 — 89	19	84.6
40 44	13	$+ 41.8$	90 — 99	16	94.3
35 39	12	$+ 38.0$	100 — 109	20	104.0
30 34	11	$+ 32.9$	110 — 119	15	114.7
25 29	13	$+ 27.2$	120 — 129	15	124.2
20 24	19	$+ 22.1$	130 — 139	14	135.2
15 19	31	$+ 16.8$	140 — 149	17	144.1
10 14	36	$+ 12.0$	150 — 159	13	154.2
$+ 5 + 9$	56	$+ 6.5$	160 — 169	34	165.7
0 $+ 4$	45	$+ 2.5$	170 — 179	17	175.6
0 — 4	65	— 2.2	180 — 189	15	183.8
— 5 — 9	73	— 6.4	190 — 199	9	195.7
10 14	44	— 11.8	200 — 209	4	205.0
15 19	33	— 16.8	210 — 219	3	212.7
20 24	23	— 21.2	220 — 229	14	224.2
25 29	9	— 26.3	230 — 239	7	237.0
30 34	12	— 32.5	240 — 249	3	246.0
35 39	17	— 37.1	250 — 259	17	255.2
40 44	8	— 42.6	260 — 269	12	263.3
45 49	3	— 46.3	270 — 279	14	275.7
50 54	3	— 53.0	280 — 289	15	283.3
55 59	6	— 56.7	290 — 299	8	293.6
60 64	2	— 60.5	300 — 309	7	303.7
65 69	2	— 68.5	310 — 319	16	315.1
70 74	—	—	320 — 329	26	323.5
75 79	—	—	330 — 339	5	332.2
80 84	2	— 80.5	340 — 349	12	345.7
— 85 — 89			350 — 359	19	355.1

were excluded, owing to the incompleteness of the material of faint magnitudes. The axes of the ellipsoid are proportional to the following three numbers:

$$a = 31 \pm 3^\circ, \quad b = 60 \pm 6^\circ, \quad c = 52 \pm 5^\circ, \quad (36)$$

in units of the mean distance r . The orientation of the ellipsoid in space is expressed by the following direction of the axes:

$$\begin{aligned} \alpha_1 &= 196^\circ, & \alpha_2 &= 231^\circ, & \alpha_3 &= 295^\circ, \\ \delta_1 &= +28^\circ.5, & \delta_2 &= -56^\circ.0, & \delta_3 &= +16^\circ.5, \end{aligned} \quad (37)$$

where the values $(\alpha_1 \delta_1)$ correspond to the shortest axis. The mean errors average $1^\circ.5$.

From the figures (36) it is seen that the ellipsoid is very nearly an ellipsoid of rotation.

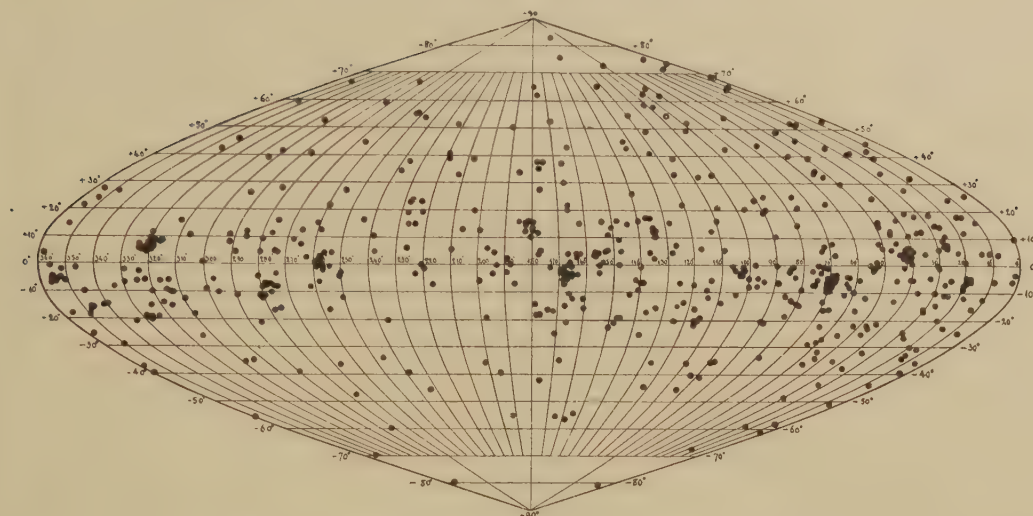


Fig. 15. Galactic distribution of the irregular variables.

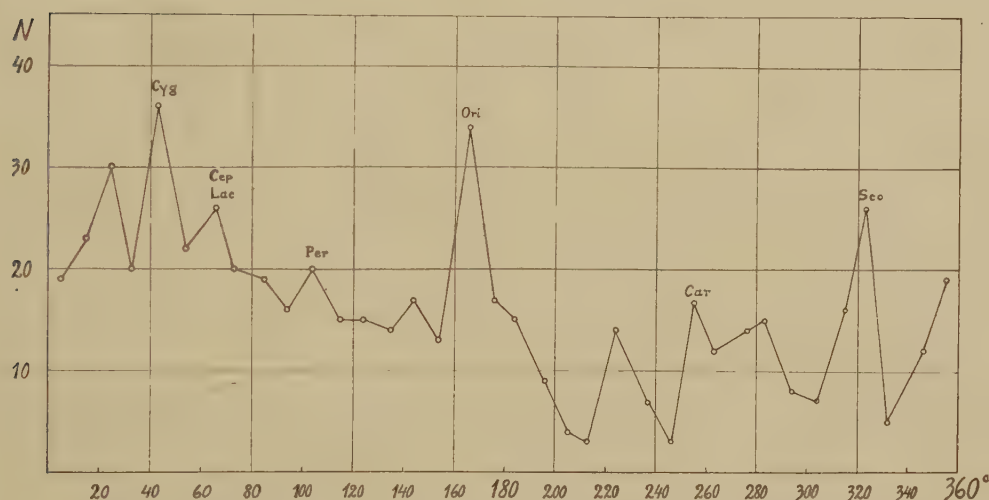


Fig. 16. Distribution of the galactic longitudes of the irregular variables.

In the following calculations the galactic coordinates, when made use of, have been taken from the Lund Observatory Tables, where the Standard pole $\alpha = 190^\circ$; $\delta = +28^\circ$ (1900.0) has been adopted. This position may not, however, represent the true pole for this particular class of stars. In the graphical representation of the z -coordinates, an error in the position of the pole will give rise to a periodic run in the mean values with galactic longitude. Such an effect has in fact been found, as will be shown below.

Although the total positive and negative z -values for the whole sky balance each other except for a slight negative excess, the mean values for the four different

quadrants show a sine-like trend of $\bar{z}'$. The curve is not symmetrical. The largest positive value is found in the third quadrant, where $\bar{z}' = +64$ for $l = 225^\circ$. The largest negative value lies in the fourth quadrant, where $\bar{z}' = -27$ for $l = 314^\circ$.

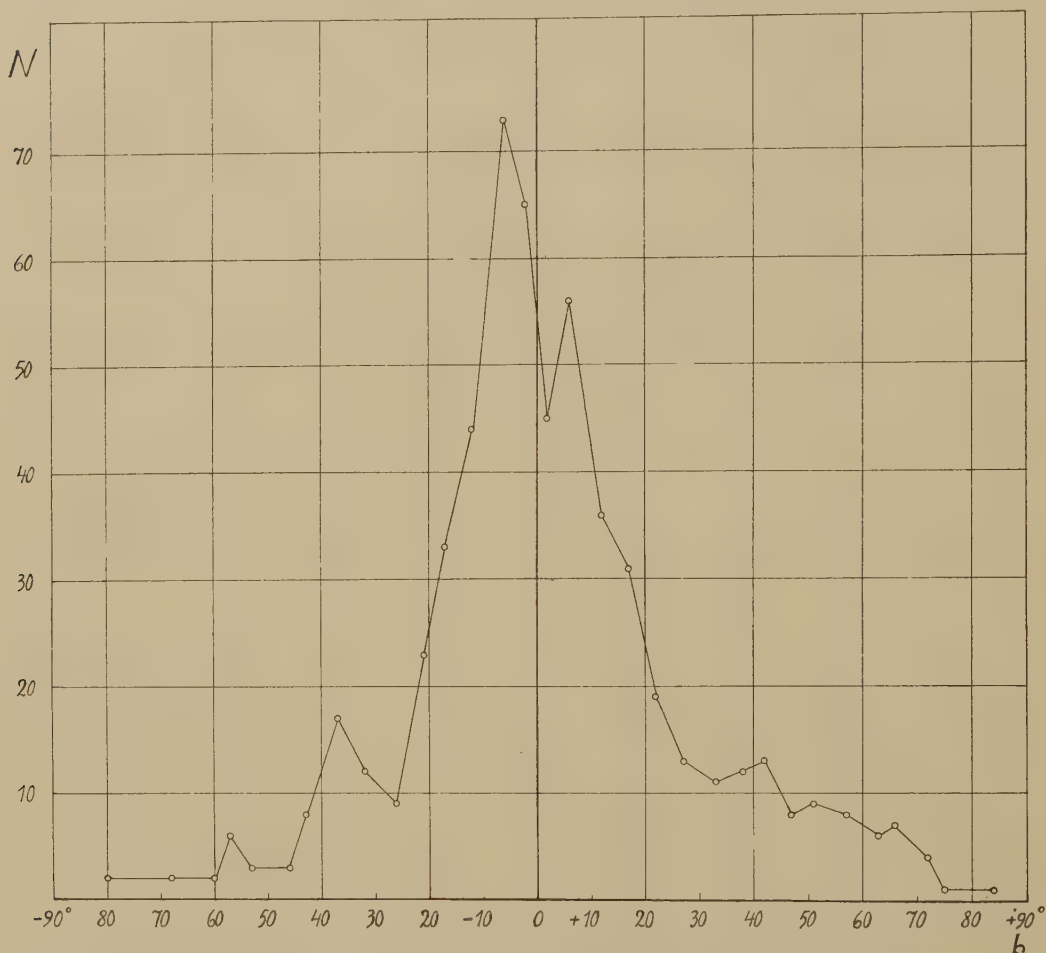


Fig. 17. Distribution of the galactic latitudes of the irregular variables.

These figures indicate a deviation of the true pole of the irregular variables from the pole of PICKERING, generally adopted as a standard.

The rectangular coordinates in the galactic system are defined by the following well known expressions:

$$\begin{aligned} x &= r \cos b \cos l, \\ y &= r \cos b \sin l, \\ z &= r \sin b, \end{aligned} \tag{38}$$

where r = the distance of the star, l, b = the galactic coordinates, obtained from the table.

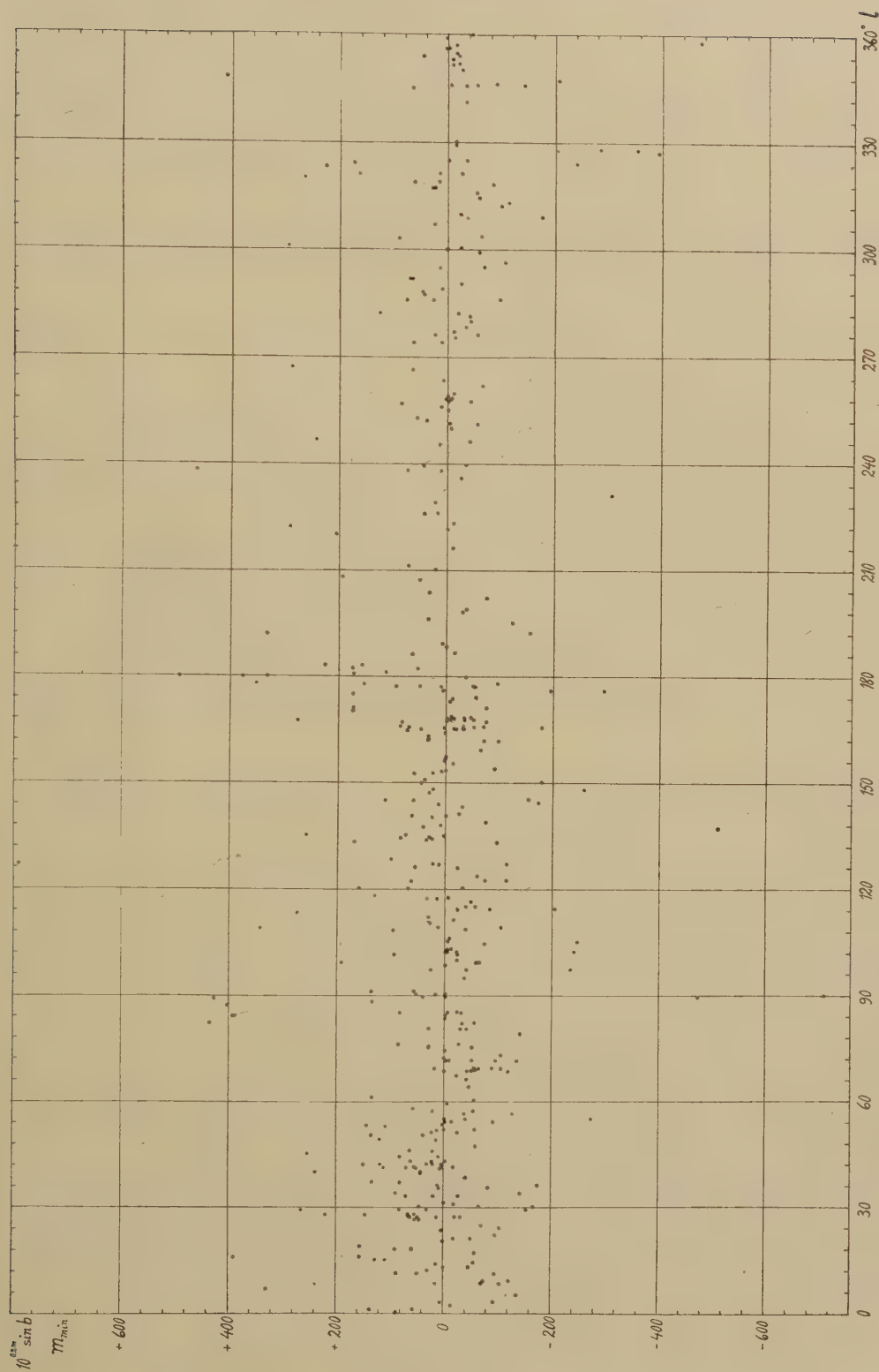


Fig. 18. Distribution of the irregular variables in galactic z' -coordinates with the galactic longitude.

Now we can write:

$$r = R 10^{0.2m}, \quad (39)$$

where $R = 10^{-0.2M}$.

M and m are the absolute and apparent magnitudes.

R represents the unit distance for stars with an absolute magnitude equal to zero.

At present R may be assumed to be a constant.

Introducing the expression for R into the formulae (38) we get:

$$\begin{aligned} x &= R 10^{0.2m} \cos b \cos l, \\ y &= R 10^{0.2m} \cos b \sin l, \\ z &= R 10^{0.2m} \sin b. \end{aligned} \quad (40)$$

In the following the distribution of z -coordinates will be discussed.

For each star has been determined the value of $z' = 10^{0.2m} \sin b$, for the maximum, medium and minimum magnitudes. These quantities are proportional to the absolute distances of the stars from the galactic plane.

When all available stars are considered ($N = 582$), the following values of the parameters of the first and the second order of the frequency-distribution of the z -coordinates have been derived. To the figures are attached the mean errors.

$$\begin{aligned} \text{Mean value:} \quad \bar{z} &= -0.1 R \pm 5.0 R, \\ \text{Average deviation: } |\bar{z}| &= 96.7 R \pm 15.8 R, \\ \text{Dispersion:} \quad \sigma_z &= 121.0 R \pm 19.8 R. \end{aligned} \quad (41)$$

The last line, giving the value of σ_z , has been derived from the computed average deviation by means of the formula:

$$\sigma = \sqrt{\frac{\pi}{2}} \vartheta.$$

From the proper motions R has been found to be 4.14 parsecs. Hence the absolute values of the parameters are the following:

$$\begin{aligned} \bar{z} &= -0.4 \pm 20.7 \text{ parsecs}, \\ |\bar{z}| &= 400.3 \pm 65.4 \quad \gg, \\ \sigma_z &= 500.9 \pm 82.0 \quad \gg. \end{aligned} \quad (42)$$

The mean value of z or the »dip».

In the above expressions (41) and (42) we observe that the mean error in $\bar{z}$ is so large that a »dip» in the galactic distribution cannot be significative. From the value obtained we should get a »dip» of -0.4 parsecs, corresponding to a position of the Sun of 0.4 parsecs above the galactic plane.

If we omit the faintest stars and use only stars for which $m_{\min} < 15.0$ we find:

$$\bar{z} = + 6.6 \pm 5.0 R \quad (N = 528). \quad (43 a)$$

For stars between the magnitudes 7.5 and 14.9 we find:

$$\bar{z} = + 6.8 \pm 5.0 R \quad (N = 511). \quad (43 b)$$

For stars with $m_{\min} < 11.5$ ($N = 255$):

$$\begin{aligned} \bar{b} &= + 3^{\circ}.4 \pm 1^{\circ}.7, \\ \bar{z} &= + 4.9 R \pm 2.9 R. \end{aligned} \quad (43 c)$$

The very bright stars which define the frequency-maximum A (Fig. 11) of the apparent magnitudes, i. e. $m_{\min} < 7.5$, give the following value of $\bar{b}$ and $\bar{z}$:

$$\begin{aligned} \bar{b} &= 0^{\circ}.0 \pm 3^{\circ}.2, \\ \bar{z} &= + 0.1 \pm 1.2 R. \end{aligned} \quad (43 d)$$

As has also been pointed out elsewhere in this paper, these stars are in fact more concentrated to the galactic plane than the fainter stars.

The $\bar{z}$ -quantities seem to assume positive values, indicating a position of the Sun *below* the plane of symmetry. Owing to the large mean errors, no definite conclusion can, however, be reached on this point.

The dispersion in z .

The dispersion in z is a measure of the thickness of the galaxy of the stars in question. This quantity has been found above (41) to be 501.9 ± 82.0 parsecs. It is of interest to compare these figures with those derived from other groups of stars. For that purpose the parameters of the galactic distribution determined by CHARLIER¹ were used. The value of σ_z for the B-stars was here found to be 59.34 ± 1.16 parsecs. The amounts obtained for the galactic dispersions of the irregular variables and the B-stars seem to satisfy the relation between σ_z and spectral class established by GYLLENBERG.²

The thickness of the system and the apparent concentration towards the galactic plane can in certain cases be used as an indicator of the distance-modulus of the stars considered. The more massive stars are on an average nearer this plane than those having less masses. Taking this fact into consideration, the irregular variables should on an average be more massive and brighter than the long-period variables, which stars are widely scattered over all galactic latitudes. They should,

¹ Lund Medd Ser II No. 34 (1926).

² Lund Medd Ser I No. 99 (1922).

however, not be expected to attain the high luminosity of the B-stars, in the mean -1^m73 , the brightest group of which, B1, has $\bar{M} = -2^m83$. From the conclusions in Chapter 11 it has been proved that a small group of irregular variables with bright apparent magnitudes appears to have a mean absolute maximum magnitude of -2^m5 .³ Further, the irregular variables in the zone of $b < \pm 15^\circ$ show an absolute maximum magnitude of -2^m16 . These groups represent the brightest stars of irregular variation. They are evidently of the same order of luminosity as the B-stars.

It can be concluded from the galactic scattering of the irregular variables that these stars also include members of low luminosity and small mass. This has also been confirmed by the investigations on the mean absolute magnitude. The mean value of the absolute magnitude for all stars is in maximum $+0^m35$. This result suggests certainly the presence of faint stars in the material. Further, the non-galactic variables ($b > \pm 15^\circ$) have been found to have at maximum the absolute magnitude $+0^m65$.

It would be of interest to carry out a similar investigation for the long-period variables, since it might be expected that this class would show a scattering in luminosity corresponding to that of the irregular variables. In fact, the period-apparent magnitude diagram of these stars, as derived by GYLLENBERG,⁴ clearly indicates the existence of different layers, as was pointed out in his paper. The difference between the maxima of the frequencies is about 2^m3 .

II. The relation between galactic position and apparent magnitude

This section will deal with the correlation between the galactic coordinates b , $\sin b$ and $10^{0.2m} \sin b$ on the one hand and apparent magnitude m on the other hand. Owing to the small amplitude of the light-variation it is of no importance which of the fundamental phases — maximum, medium, or minimum — is used as variate. As a rule the general computations have been carried out for at least two phases, while the more detailed investigations in the following paragraphs have been carried out only for the minimum magnitudes.

The material has been treated with regard to the following points of view: —

1. Construction of scattering diagrams, using:
 - a. Apparent coordinates: $z = \sin b$,
 - b. Reduced coordinates: $z' = 10^{0.2m} \sin b$.
2. Statistical treatment of the material.

³ The absolute magnitudes derived in the following are not corrected according to 1) absorption in space, 2) effect of correlation between absolute and apparent magnitude.

⁴ Lund Medd Ser II No. 54 (1930).

1. Construction of scattering diagrams.

a. The apparent coordinates: $z = b$ and $z = \sin b$.

The average values of $\sin b$, plotted against the minimum magnitudes, show a fairly large dispersion. For stars brighter than $7^m.00$ the mean value of $|\sin b|$ does not, however, exceed 0.500. From $7^m.50$ onward the dispersion suddenly increases. There is possibly to be noted an indication of three or four preferential strata of stars, although faintly pronounced.

b. Reduced coordinates: $z' = 10^{0.2m} \sin b$.

The scattering diagrams representing the correlation between z' and m have been constructed for the maximum and minimum magnitudes m . The result is a peculiarly defined distribution, reproduced in Fig. 19 and 20. Since the above coordinates are quantities which are proportional to the true distances of the stars from the galactic plane (assuming R to be a constant), the expected distribution would be represented by a zone of stars around the Milky Way plane. In fact, the z' -coordinates, instead of showing constant average values, increase systematically with m .

In the graphical distribution may be observed two or three »arms», or groups, symmetrically diverging from the galactic plane. These arms appear as concentrations of stars, separated by more or less empty spaces. These different layers might be assumed to consist of stars distinguished by different absolute magnitudes. The absolutely brightest stars appear close to the galactic plane. The layers might therefore represent groups of different absolute luminosities of the irregular variables.

2. Statistical treatment.

a. Apparent coordinates.

The mean values and the average values of b and $\sin b$ were formed within intervals of $0^m.5$ (Table 10). The distributions in Fig. 21 were obtained. The calculations were carried out for medium and minimum magnitudes. It may be observed that the difference in scattering in an array is small in both phases. It seems, in fact, that the minimum-phase shows the smallest scattering.

The empirical curve drawn through the mean or average values of $\sin b$ appears in its general features to agree with a theoretical one. Let us regard a surface-element on the sphere, where the z -axis points towards the galactic pole.

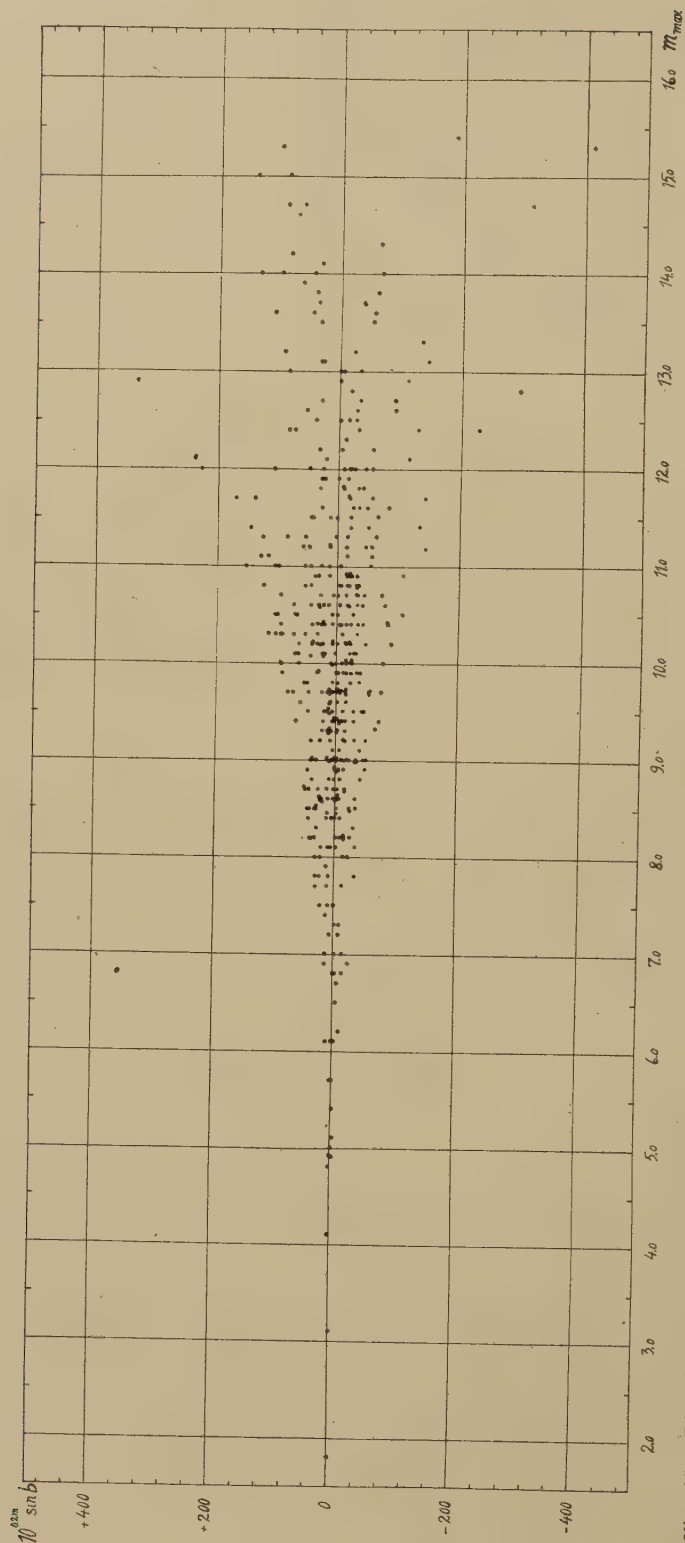


Fig. 19. Distribution of the reduced galactic z' -coordinates $100.2m \sin b$ of the irregular variables with the apparent maximum magnitudes.

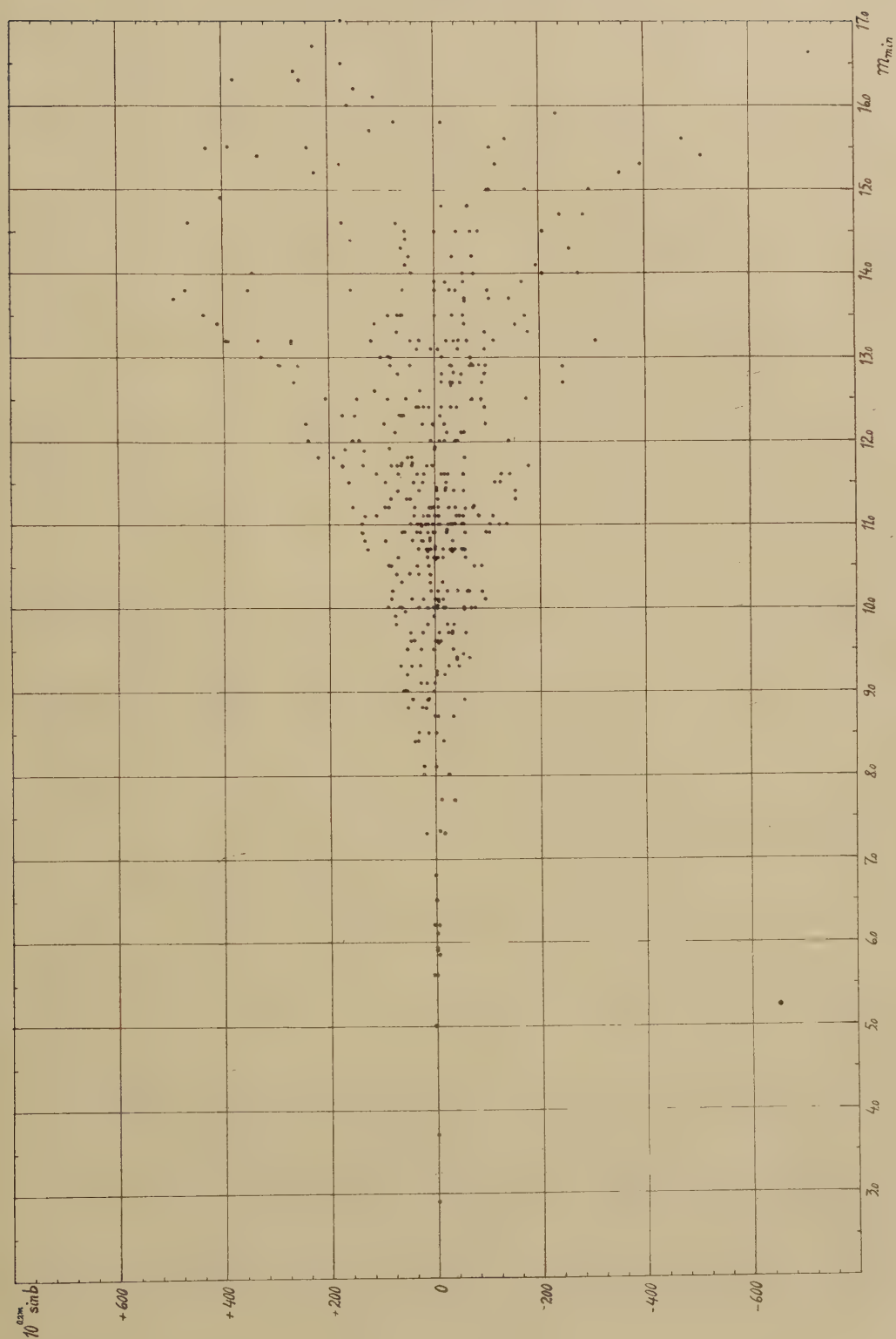


Fig. 20. Distribution of the reduced galactic z' -coordinates $100.2m \sin b$ of the irregular variables with regard to the apparent *minimum* magnitudes.

TABLE 10.

Correlation between apparent *minimum* magnitude and galactic *z*-coordinates.

$m_{(\min)}$	N	$\bar{m}$	$\bar{b}$	$ \bar{b} $	$\sigma(\bar{b})$	$ \overline{\sin b} $	$\overline{10^{0.2m} \sin b}$	$ \overline{10^{0.2m} \sin b} $	σ
2.5—2.9	1	2.90	— 8°0	8°0		.139	— 0.6	0.6	
3.0—3.4	—	—	—	—		—	—	—	
3.5—3.9	1	3.70	— 6.0	6.0		.105	— 0.5	0.5	
4.0—4.4	—	—	—	—		—	—	—	
4.5—4.9	1	4.60	— 29.0	29.0		.485	— 3.9	3.9	
5.0—5.4	1	5.00	+ 28.0	28.0		.469	+ 4.7	4.7	
5.5—5.9	5	5.76	+ 3.2	10.0	$11^{\circ}8 \pm 3^{\circ}7$	.171	+ 0.6	2.3	2.8 ± 0.9
6.0—6.4	3	6.17	— 1.3	12.7	15.5	.218	— 0.4	3.7	4.6 1.9
6.5—6.9	2	6.65	+ 5.5	5.5	1.9	.096	+ 2.1	2.1	1.3 0.6
7.0—7.4	3	7.30	— 2.7	31.3	38.0	.510	— 1.6	14.8	17.5 7.2
7.5—7.9	2	7.70	— 48.5	48.5	40.7	.632	— 22.1	22.1	15.7 7.8
8.0—8.4	8	8.21	+ 16.1	29.9	35.2	.473	+ 11.5	21.0	24.4 6.1
8.5—8.9	12	8.74	+ 12.7	30.9	32.7	.471	+ 11.4	26.7	28.3 5.8
9.0—9.4	22	9.22	+ 8.7	31.2	40.4	.472	+ 5.9	32.8	41.6 6.3
9.5—9.9	28	9.70	+ 6.2	23.2	29.3	.373	+ 8.2	32.5	41.0 5.5
10.0—10.4	42	10.15	+ 7.6	23.9	29.7	.378	+ 13.2	40.6	50.1 5.5
10.5—10.9	60	10.73	+ 2.1	17.6	22.3	.283	+ 3.1	39.6	50.0 4.6
11.0—11.4	64	11.16	— 2.1	17.9	22.2	.288	— 6.0	48.7	60.4 5.3
11.5—11.9	50	11.67	+ 5.2	24.1	29.6	.379	+ 19.5	82.3	101.4 10.1
12.0—12.4	44	12.18	+ 2.9	17.4	22.4	.281	+ 8.8	76.5	97.2 10.4
12.5—12.9	45	12.72	+ 1.8	18.6	24.0	.305	+ 7.2	106.0	135.4 14.3
13.0—13.4	44	13.18	+ 4.9	16.7	21.5	.271	+ 30.2	117.9	151.2 16.1
13.5—13.9	38	13.69	— 2.4	15.3	18.5	.245	— 26.5	135.3	160.5 18.4
14.0—14.4	28	14.12	— 3.5	12.5	15.4	.212	— 37.2	141.8	174.5 23.3
14.5—14.9	24	14.65	+ 5.6	14.2	18.8	.225	+ 67.4	191.4	248.0 35.8
15.0—15.4	15	15.22	— 4.3	12.4	14.5	.214	— 79.7	237.7	277.8 50.7
15.5—15.9	15	15.63	— 4.7	12.3	14.2	.206	— 108.3	275.6	318.8 58.2
16.0—16.4	17	16.20	— 3.5	13.2	17.2	.218	— 83.1	375.9	489.5 83.9
16.5—16.9	7	16.56	+ 2.6	8.6	8.1	.148	+ 97.1	301.1	290.6 77.7

The variate representing the coordinate in the galactic plane may be called φ . The mean value of $\sin b$ has the following expression:

$$\overline{\sin b} = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} db d\varphi \sin b \cos b : \int_0^{\frac{\pi}{2}} \int_0^{2\pi} db d\varphi \cos b,$$

where $db d\varphi \cos b$ denotes the surface-element.

Hence we obtain:

$$\overline{\sin b} = \frac{1}{2}. \quad (44)$$

This is the largest mean value obtainable. On account of the flatness of the star-system we do not observe this maximum-value of $\overline{\sin b}$, but as may be expected we obtain a curve that tends asymptotically towards $\overline{\sin b} = \frac{1}{2}$ for its brightest magnitudes. For faint magnitudes $\overline{\sin b}$ diminishes.

For the apparently bright magnitudes there is a distinct indication of the presence of a secondary group, with a value of $\overline{\sin b}$ fairly well separated from

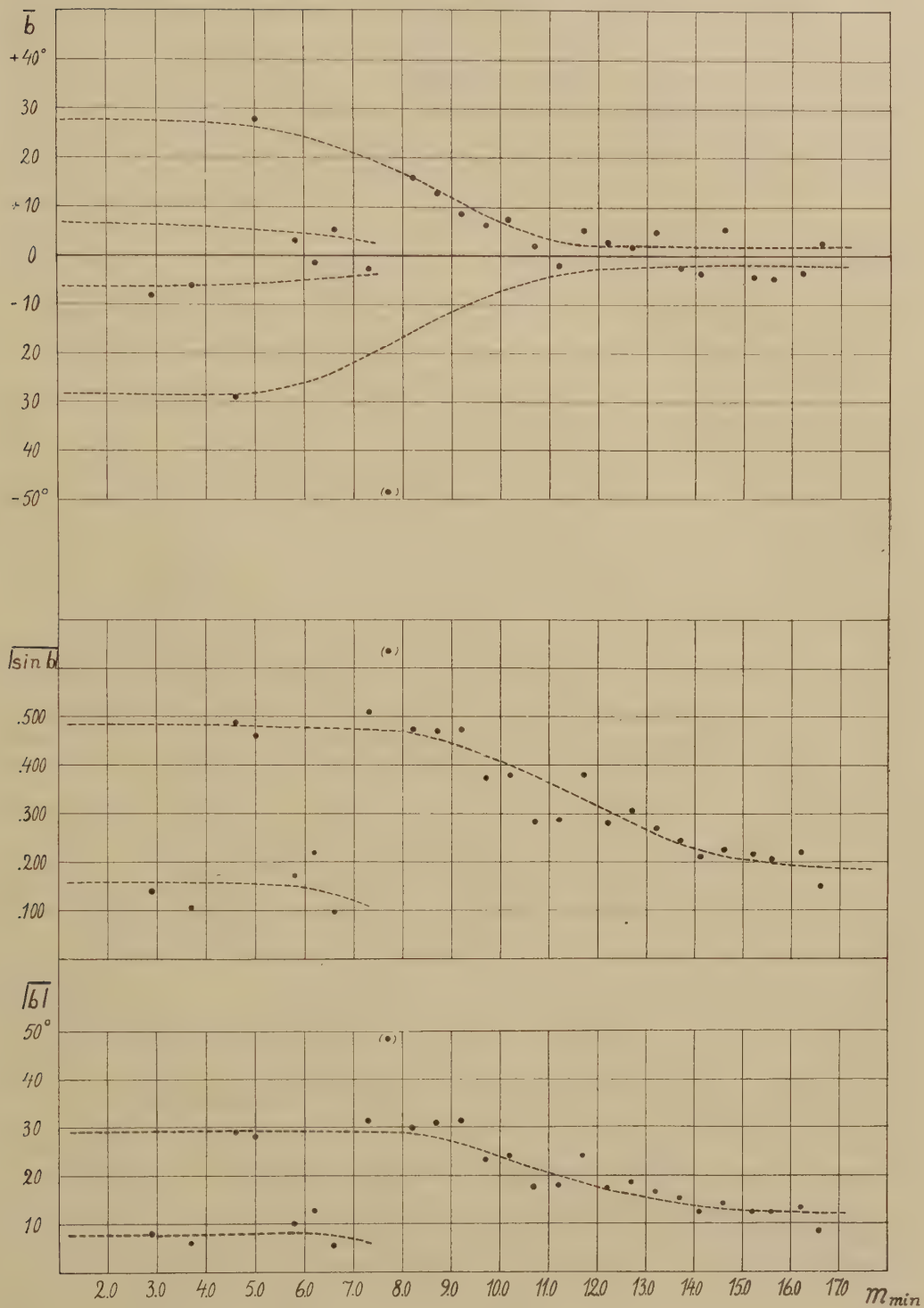


Fig. 21. The correlation between the apparent distance from the galactic plane and the apparent *minimum* magnitude.

that of the main group. The curves have been drawn tentatively to fit in best with the dots.

In order to investigate whether the stars at the galactic centre influence the distribution in a certain way, the stars up to 60° on both sides of the galactic centre (assumed at $l=325^\circ$) have been excluded, and the same computations carried out (Table 11). No difference is, however, observed in the two cases.

TABLE 11.

The quantities $|\sin b|$ and $|10^{0.2m} \sin b|$ correlated to $m_{\min}$. [Stars within 60° of the galactic centre excluded.]

$m_{\min}$	n_1	$\bar{m}_{\min}$	$ \overline{\sin b} $	$ \overline{10^{0.2m} \sin b} $
2.5—2.9	1	2.90	0.1390	0.60
3.0—3.4	—	—	—	—
3.5—3.9	1	3.70	0.1050	0.50
4.0—4.4	—	—	—	—
4.5—4.9	1	4.60	0.4850	3.90
5.0—5.4	—	—	—	—
5.5—5.9	3	5.87	0.1150	1.67
6.0—6.4	3	6.17	0.2180	3.73
6.5—6.9	2	6.65	0.0960	2.10
7.0—7.4	3	7.30	0.5097	14.80
7.5—7.9	1	7.70	0.9880	34.60
8.0—8.4	6	8.22	0.4832	21.83
8.5—8.9	9	8.74	0.4720	26.4
9.0—9.4	15	9.22	0.4215	29.1
9.5—9.9	20	9.65	0.3918	33.2
10.0—10.4	31	10.15	0.3476	37.3
10.5—10.9	45	10.73	0.2294	31.7
11.0—11.4	42	11.17	0.3095	53.0
11.5—11.9	40	11.68	0.3598	78.3
12.0—12.4	30	12.20	0.2481	67.6
12.5—12.9	32	12.73	0.3287	112.6
13.0—13.4	33	13.18	0.2473	108.1
13.5—13.9	23	13.70	0.2139	117.0
14.0—14.4	20	14.12	0.1932	128.6
14.5—14.9	16	14.66	0.2354	201.9
15.0—15.4	10	15.22	0.1913	214.1
15.5—15.9	12	15.64	0.2147	289.0
16.0—16.4	5	16.18	0.1790	303.8
16.5—16.9	1	16.50	0.3580	714.0
All	405			

b. The average value of the z' -coordinates, correlated to apparent magnitudes.

In a previous paragraph the individual z' -values have been plotted against the apparent magnitudes as abscissa. The distribution thus obtained shows a very peculiar appearance. Since the apparent magnitudes may be considered to represent the distances of the stars, it is to be expected that the z' -values would be distributed round the galactic plane, with a constant scattering with increasing distance.

In order to study the distribution closer, the average values of z' , or $10^{0.2m} \sin b$, were formed for intervals of 0^m.5. The calculations refer to medium as well as to minimum magnitudes. In the following discussion the minimum magnitude-curve has been used. There seems to be no need to repeat the investigations for all three phases.

We will now consider the construction of the curves of $\bar{z}'$, with reference to minimum of light.

Since z' for increasing m is expected to run parallel to the m -axis, different sources which might explain the peculiar deviations in the trend of the curve have been considered.

As regards the linear values of the stars' distances from the galactic plane:

$$z = R 10^{0.2m} \sin b,$$

or:

$$z = R z', \quad (45)$$

there are two possibilities for a change in the mean value of z with increasing distances like that shown in the figure.

The first possibility is a systematic correction in R , depending on r or m . The second possibility is a systematic effect in m . The effect observed shows that $\sin b$ appears to be multiplied by a factor systematically increasing with increasing magnitude or increasing distance. This would occur if the magnitudes for some reason were measured too faint, with an amount that increases with increasing m . A systematic effect in R is, in fact, more difficult to analyse, in so far as it depends on the correlation between M and m . We shall return to that in more detail below.

But there is yet another explanation, namely a general absorption of light in space. Since the effect of this phenomenon depends on the distance, absorption would evidently enter here, and account for part of the systematical errors in the apparent magnitudes.

Of course the irregular variables, belonging to the absolutely brightest stars known and hence relatively remote, should furnish a very good material for determining the absorption in space.

At present we will consider the problem from the following points of view: —

1. It was attempted to reduce the observed curve to a line parallel to the m -axis by assuming different coefficients of absorption, α .
By m_α we denote the magnitudes affected by absorption.
2. The theoretical curve (the straight line) was reduced by means of different coefficients of absorption to curves similar to the one obtained from the observations.

1. Reduction of the observed curve to a straight line parallel to the axis.

The method used was as follows: —

Starting from the »correct» values of m — i. e. the values of the magnitudes assuming no absorption — with their respective computed distances, the effect of the absorption was added to these magnitudes. Then the m_a -values are obtained. Hence, if we put these magnitudes corrected for absorption identical with the observed ones, the $\sin b$'s were read from the observed curve, m_a being used as argument. Then the values of $10^{0.2m_a} \sin b$ were formed. Now, if we had used the correct factor, $10^{0.2m_a}$, the values of the product $10^{0.2m_a} \sin b$ would reduce themselves to represent a line parallel to the m -axis, otherwise not.

The calculations were performed for the following coefficients of absorption: 0^m5, 1^m0, 1^m5, 2^m0, 3^m0 per 1000 parsecs. Further, a constant value of R was preliminarily assumed.

It will be seen from Fig. 22 that a coefficient of absorption of 3^m is required to get the curve approximately straightened out. In Fig. 22 the abscissa represents the »false» magnitudes. It is necessary to refer the curve to these magnitudes, since they are the ones available (= observed). When the coefficient of absorption is known, however, it will be possible to determine the correct magnitudes.

The effect observed in the present set of observations seems in the mean to be equivalent to that of a constant spatial absorption of about 3^m.

This value of absorption is, however, too large to be accepted.

It may be remarked that the part of the curve before 7^m00 is not reduced to the same straight line as the remainder of the curve. Therefore a difference in R is suggested, possibly with a leap at $m = 7^m00$.

2. Reductions of a theoretical curve to fit the observations.

As in the previous case, the »correct» values of m , i. e. the values of the magnitudes with no absorption present, were taken as the starting point. The correction for absorption was added as before, and the m_a were obtained. By the use of m_a as argument the values of $10^{0.2m_a}$ were deduced.

We consider the expression:

$$z = R 10^{0.2m} \sin b. \quad (46)$$

Supposing $\frac{z}{R}$ to be a constant, the expression is transformed to:

$$\sin b = \frac{\text{const}}{10^{0.2m}}. \quad (47)$$

For the sake of convenience the constant was put equal to unity. Hence we obtain the values of $10^{0.2m_a} \sin b$, which should entirely fit the observed curve, if

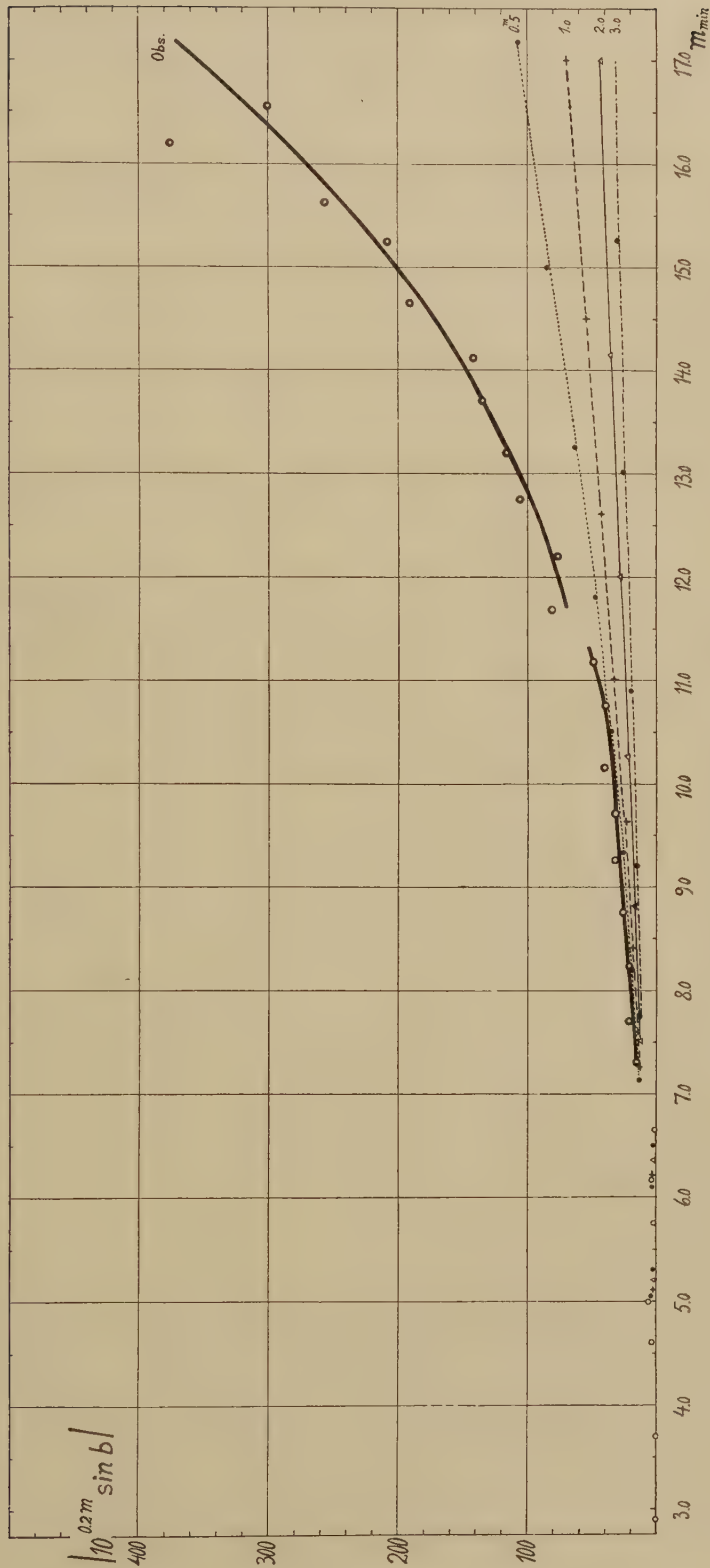


Fig. 22. Reduction of the z' -values, derived from the observations, to a line parallel to the m -axis, by means of different assumptions concerning the light-absorption.

the correct magnitudes m were found. Then the two scales would coincide, and the trend of the two curves would be parallel. In each case we obviously have to regard the magnitudes m_a as identical with the observed ones.

This reduction was carried out for the following values of the coefficient of absorption, when R was assumed to be constant:

$$a = 1^m, 1^m5, 2^m, 3^m, 4^m, 5^m \text{ per } 1000 \text{ parsecs.}$$

From this was derived the *family of curves* represented in Fig. 23.

In this diagram the constant $\frac{z}{R}$ has been arbitrarily chosen equal to 20, in order to make the left end of the curve fit as closely as possible the mean value of the observed curve at 8^m0 — 9^m0 . This first part of the curve, A , is at present left out of account.

The curve seems to indicate a higher absorption in its last part than in the part from 7^m00 to 11^m00 . This may be due to the fact that a variation in R is disregarded.

In order to study the effect more closely, the same procedure as above has been carried out, but with *different values of R for each value of absorption*.

Each value of the coefficient of absorption thus gives rise to a *family of curves* depending on R . Thus the diagrams in Fig. 24 are obtained.

It is of interest to observe the indication, mentioned above, of a larger absorption in the part of the curve below 11^m0 . The curvature is here consistent with an absorption of 3^m per 1000 parsecs, while the part 7^m00 — 11^m00 fits the 2^m -curve.

Now I have called the effect in the z' -values »absorption». As shall be shown below, the effect is probably also due to other causes. In fact, the run of at least part B of the curve may be fully explained by another effect. Here only the presence of an effect that is *equal* in its amount to an absorption in space has been shown.

Now it might be presumed that the stars near the galactic centre influence the material and thus systematically falsify the results. In order to eliminate such a possible source of error, the stars in longitudes $\pm 60^\circ$ from the galactic centre were excluded. The same calculations of $\bar{z}'$ and $\sin \bar{b}$ were carried out, and the values obtained have been entered into the diagrams. There is found no difference in the general run of the two sets of values.

a. Part B of the curve [$m_{\min} = 7.00$ — 11.25].

The part of the curve representing the observations between the magnitudes 7.00 — 11.25 can be regarded as approximately a straight line. The supposition suggests itself that the increase of the ordinate is caused by the correlation between absolute and apparent magnitude, since this correlation runs in the same direction. We shall now investigate the effect of this correlation. At another place in this

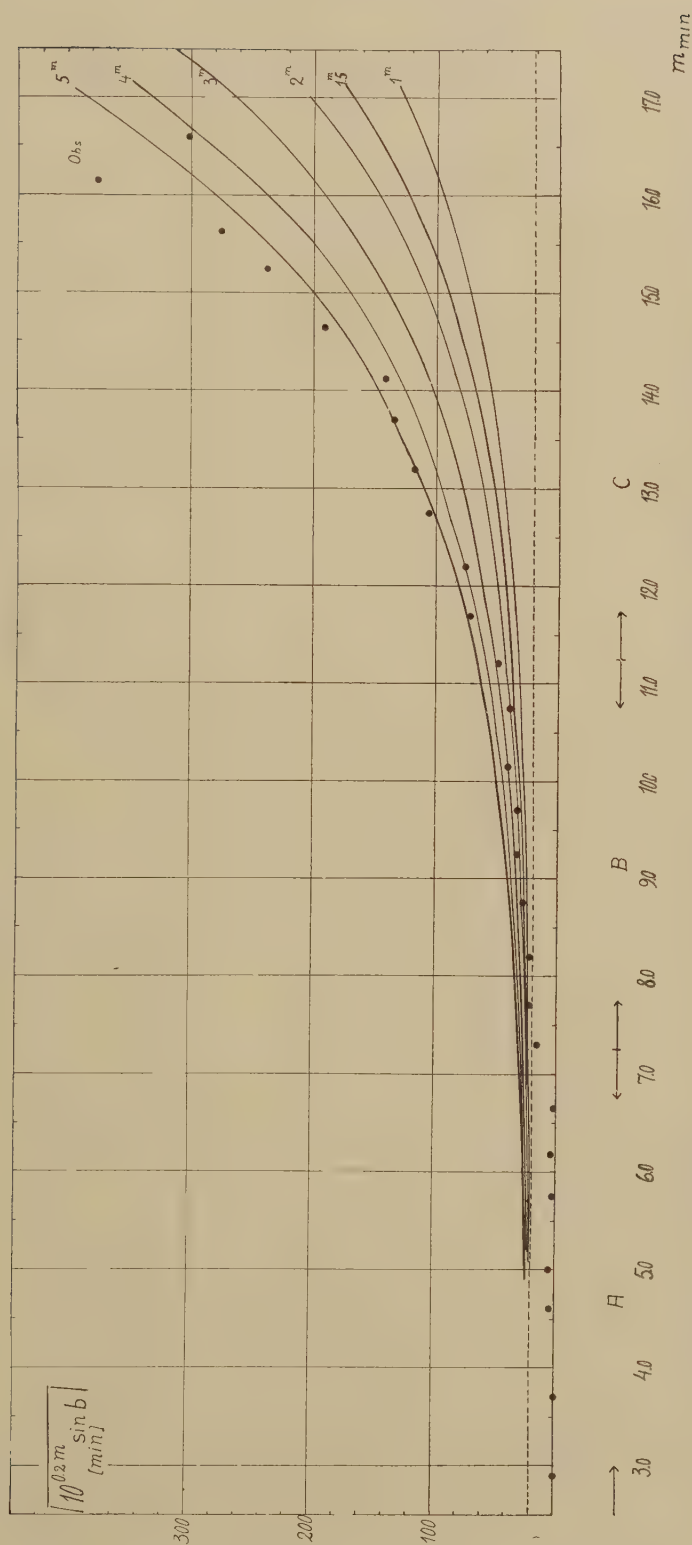


Fig. 23. The curves give the theoretical average value of z with each class of apparent magnitude for different values of the absorption in space. R is regarded as a constant. The dots are the actually observed values of z in the present material.

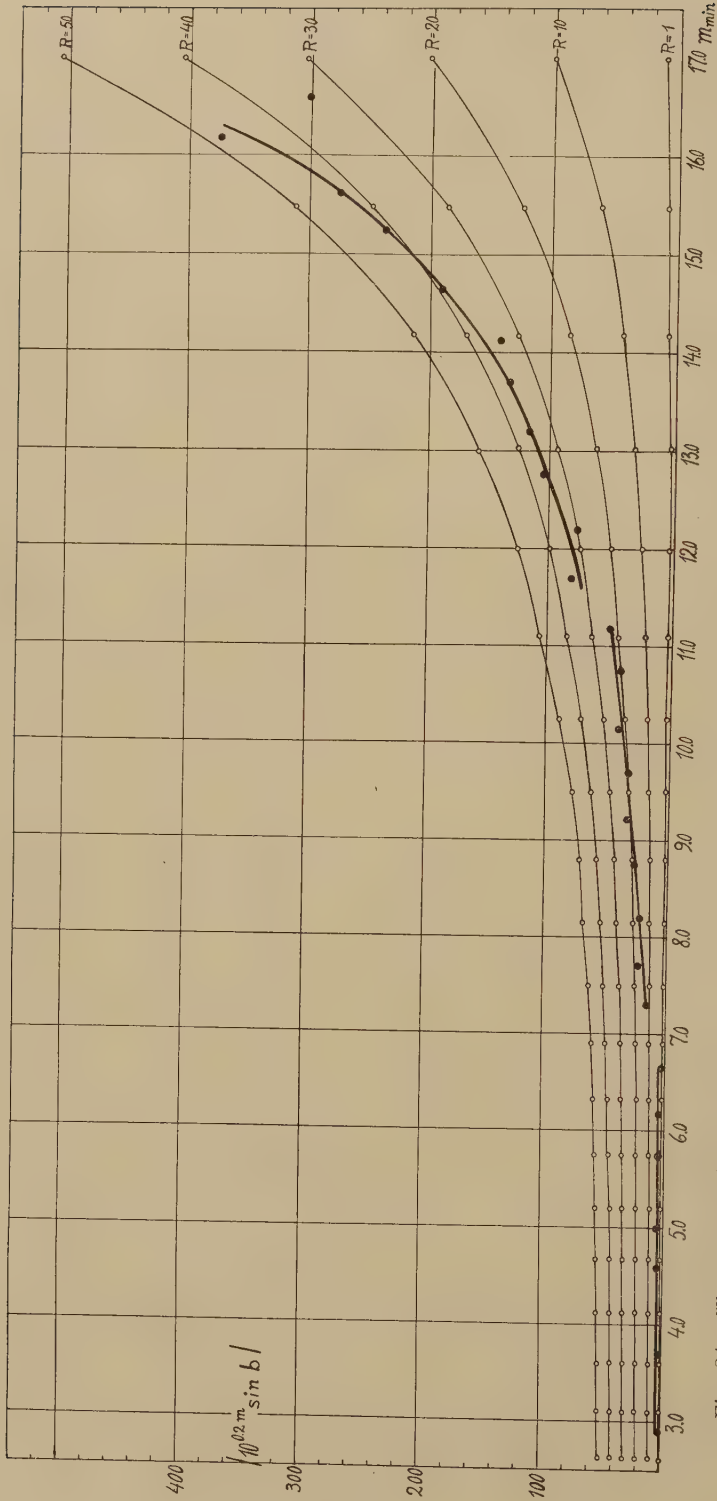


Fig. 24. The curves give theoretical average value of $\bar{m}_z$ for each class of apparent magnitude with a constant value of the absorption in space [here the value $2m$ per 1000 parsecs is chosen], and different values of R . The dots represent the values obtained from the observations.

paper this question has been discussed, and a maximum and a minimum value of the coefficient of correlation have been derived.

The regression-line to be considered is written:

$$M - M_0 = r_{M,m} \frac{\sigma_M}{\sigma_m} (m - m_0). \quad (48)$$

The function representing the correlation between M and m can be assumed to have the following form:

$$\varphi(M, m) = \frac{\text{const}}{\sigma_M \sigma_m 2\pi \sqrt{1 - r_{M,m}^2}} e^{-\frac{1}{2}f}, \quad (49)$$

where:

$$f = \frac{1}{1 - r_{M,m}^2} \left[\frac{(M - M_0)^2}{\sigma_M^2} - 2r_{M,m} \frac{(M - M_0)(m - m_0)}{\sigma_M \sigma_m} + \frac{(m - m_0)^2}{\sigma_m^2} \right].$$

Now it can easily be shown⁵ that the coefficient of correlation $r_{M,m}$ is defined by:

$$r_{M,m} = \frac{\sigma_M}{\sigma_m}. \quad (50)$$

Hence the line of regression assumes the following form:

$$M - M_0 = r_{M,m}^2 (m - m_0),$$

or if m_2 and m_1 are two values of the apparent magnitudes:

$$M_2 - M_1 = r_{M,m}^2 (m_2 - m_1). \quad (51)$$

In the latter formula we put: $m_2 - m_1 = 4.25$.

Now we may regard the effect in absolute magnitude on the basis of the following values of $r_{M,m}$:

$$\begin{aligned} 1) \quad r_{M,m} &= +0.82, \\ 2) \quad r_{M,m} &= +0.77, \\ 3) \quad r_{M,m} &= +0.62, \\ 4) \quad r_{M,m} &= +0.43. \end{aligned} \quad (52)$$

$$1. \text{ Here: } M_2 = M_1 + 2.86. \quad (53)$$

From the absolute magnitude we obtain the value of R as follows:

$$R = 10^{-0.2M},$$

$$\text{Hence: } R_2 = 10^{-0.2(M_1 + 2.86)}, \quad (54)$$

$$\text{or: } R_2 = R_1 10^{-0.572}.$$

$$\text{Putting: } R_1 = R_2 \gamma, \quad (55)$$

$$\text{then: } \gamma = 3.73. \quad (56)$$

$$\text{Hence: } R_1 = 3.73 R_2. \quad (57)$$

⁵ GYLLENBERG: Lund Medd Ser I No. 142 (1935). See also note p. 96.

The z' -coordinates of the apparently brightest stars should thus be multiplied by the factor 3.73, or the faintest stars should be multiplied by a factor $\frac{1}{3.73}$, in order to obtain agreeing z' -values.

We may now regard the effect on the factor γ caused by a difference in the coefficient of correlation:

$$2. \quad M_2 = M_1 + 2.52, \quad (58)$$

$$R_1 = 3.19 R_2. \quad (59)$$

$$3. \quad M_2 = M_1 + 1.63, \quad (60)$$

$$R_1 = 2.12 R_2. \quad (61)$$

$$4. \quad M_2 = M_1 + 0.79, \quad (62)$$

$$R_1 = 1.44 R_2. \quad (63)$$

The last case must be regarded as the minimum value of the effect of the correlation, the first case as the maximum value.

In order to see how large a correction is required to reduce the observed curve to a straight line with a constant mean z' -coordinate, we determine the values of z' from readings of the curve.

$$\begin{aligned} m_1 &= 7.00; & 10^{0.2m} \sin b_1 &= 14, \\ m_2 &= 11.25; & 10^{0.2m} \sin b_2 &= 47. \end{aligned} \quad (64)$$

Now we have:

$$z_1 = R_1 10^{0.2m} \sin b_1 \quad \text{and} \quad z_2 = R_2 10^{0.2m} \sin b_2.$$

Then:

$$\frac{z_1}{z_2} = \frac{14}{47} \cdot \frac{R_1}{R_2}. \quad (65)$$

Now since the condition of solution is $z_1 = z_2$, we obtain:

$$R_1 = 3.36 R_2. \quad (66)$$

The reduced apparent galactic coordinates of stars of magnitude 7.00 should therefore be multiplied by the factor 3.36 for the mean z -coordinate to be equal to that of 11^m25.

Thus the correlation between M and m would alone be able to account for the average increase in the z' -values, provided the coefficient of correlation is as large as + 0.79. The value of this coefficient is a comparatively large one, and it is only acceptable on certain conditions.

A value of $r_{M,m}$ of this order of magnitude has in fact been obtained by means of the absolute magnitudes as derived from the proper motions (see Chapter 11). These solutions give a mean value of $r_{M,m}$ equal to $+0.737$.

If such a large value of the coefficient $r_{M,m}$ is real, it is possibly an indication of the existence of different groups of irregular variables with different mean absolute magnitudes, a kind of giant-dwarf-groups. If this is true, a certain part of the numerical value of the above given coefficient of correlation represents a false correlation.

It will be shown below that there are further reasons to accept the possibility of an existence of such groups within the class of irregular variables.

From the above considerations it would thus be possible to explain the appearance of this part of the curve without any need to accept the existence of an absorption in space.

However, the above explanation cannot be a complete one, because there is no reason to assume that space is completely free from absorption. The constant coefficient of absorption may be very small, but there are still the different clouds of absorbing matter to be considered. According to an investigation by JESSE L. GREENSTEIN⁶ the effect of the absorbing clouds on the star-light is the same as that of continuously distributed matter.

The correct explanation of the peculiarity of the curve is therefore with the greatest probability a summary effect of the absorption and the correlation between M and m . Probably the latter effect is the dominant one.

The present material does not permit a well-defined separation of the two effects, however. Such a separation would most conveniently be performed provided the coefficient of absorption were known from an independent method, or the real distances of the stars in the group were known, or, alternatively, the true value of the coefficient of correlation $r_{M,m}$.

The problem might also be regarded in the opposite way. We might ask what value of the coefficient of absorption would be required to cause the observed effect. If the curvature alone is regarded, we should from the general trend of the curve conclude an absorption of between 2 and 3 magnitudes per 1000 parsecs (see Fig. 23). From the curve the correction to m can easily be calculated.

We have:

$$\begin{aligned} z_1' &= 14, \\ z_2' &= 47, \end{aligned} \tag{67}$$

and we want to determine the correction needed for m_2 to reduce z_2' to z_1' .

R may be regarded as a constant. Then:

$$z_2' = R 10^{0.2m_2} \sin b_2 = 14. \tag{68}$$

⁶ Harvard Tercentenary Paper No. 17 (1938).

From the observations we have: $\sin b_2 = 0.320$.

Hence:

$$\begin{aligned} 10^{0.2m_2} 0.320 &= 14, \\ \text{or: } 10^{0.2m_2} &= 43.7, \end{aligned} \tag{69}$$

which corresponds to a value of $m_2 = 8.20$.

The observed magnitude 11.25 should then need a correction of 3^m05.

Assuming the distances defined by the apparent magnitudes to be correct, we should put the magnitude 11.25 at about 1775 parsecs and 7.00 at 260 parsecs. This would give an absorption of 2^m0 per 1000 parsecs.

This coefficient of absorption agrees with that obtained by BOTTLINGER and SCHNELLER.⁷ The methods used do not differ in principle from each other. In the case of the Cepheids, as first pointed out by GYLLENBERG,⁸ a large false absorbing effect may be introduced by assuming an erroneous galactic pole, since the Cepheids form a very concentrated belt round the galactic plane.

In the case of the irregular variables — which have a rather wide galactic distribution — this effect is inconsiderable and can be neglected.

It was shown by GYLLENBERG that the absorption derived from the material of the Cepheids could possibly be accounted for to the whole of its amount by the mentioned false definition of the pole and the effect of the correlation existing between M and m . There is therefore every reason to suppose the constant galactic absorption to be smaller than 2 magnitudes.

From the point of view of these considerations it is consequently reasonable to suppose the present effect in the distribution of irregular variables referred to the galactic plane to be caused by an addition of 1) the effect of the correlation between M and m and 2) the absorption in space.

b. Part A of the curve [$m < 7.00$].

Above 7^m the z' -coordinates show a small increase. Since the stars of these magnitudes are fairly near to us, an absorption will at first be neglected here. The only effect to be considered is that of the correlation between M and m . The values given in Table 9 clearly show an increase in the z' -values that can be explained as an effect of the correlation mentioned.

From the discussions in other chapters it is evident that these stars form a group of their own among the irregular variables. They are distinguished in the frequency-curve of the apparent magnitudes as maximum A. The determinations of the absolute magnitudes from the proper motions show them to be far brighter than the main group, B. The difference in absolute magnitudes is about 5 magnitudes.

⁷ ZfA No. 1, p. 339 (1930).

⁸ Lund Medd Ser I No. 144 (1936).

This difference in absolute brightness is consistent with a ratio between the values of R of 10, viz. the stars of group A have a value of R that is 10 times as large that of group B .

A sudden change in R at the 7th magnitude is therefore to be presumed.

c. **Part C of the curve [$m > 11.25$].**

Below 11^m there is to be noted another discontinuity in the curve. Evidently either there is a new leap in R , or there enters at this point a larger absorption, which might be assumed to be caused by the presence of absorbing clouds or matter at this distance. Probably both alternatives can occur simultaneously.

At present, with the small material available, it is not possible to distinguish the two effects. Since no proper motions are known for the stars of these faint magnitudes, the value of R cannot be determined in the same way as has been done for the brighter magnitudes. Further, it is not known whether an exceptionally large absorption is present for stars of 11^m . Here we only put forward the possibilities that might explain the discussed phenomenon.

While the increase in the z' -values for group B can be attributed exclusively to the correlation between absolute and apparent magnitude, though, as the above discussion made clear, it is more probable to assume a composite effect of absorption and correlation, it is evident that a simple explanation cannot satisfy the curve after 11th magnitude. Allowing for a leap in R , which probably exists, the trend of the curve is here different from the rest of the curve. It might of course be possible that the regression of M and m is not linear, and this *might* then explain the trend of the far end of the curve.

For distances corresponding to these magnitudes, however, the absorption cannot be neglected. If the absorption alone is considered to be the cause of the increase in z' , we find that in Fig. 22, a representation of the *family of curves* for an absorption of 3^m per 1000 parsecs satisfies the curvature of the observed curve. The total effect in the z' -coordinates may therefore be considered to *be equal to* that of an absorption of 3^m per 1000 parsecs. Since it is very improbable that the absorption alone should be responsible for this effect, and since it has been proved above that the correlation between M and m accounts for a considerable part of the observed effect, it is to be assumed that the third group C is subject to the same three effects as group B :

1. correlation between M and m ,
 2. difference in mean value of R ,
 3. light-absorption.
- (70)

As before it is not possible to distinguish the different effects.

Here might also enter a fourth effect, which can be neglected in the earlier groups: viz. an error in the observed magnitudes. The faint magnitudes might

possibly be observed systematically too faint and thus contain an error that depends on m . Such an error would, of course, give rise to exactly the observed effect.

The individual stars plotted against the magnitudes show the existence of several arms or streams, emerging from the main branch near the galactic plane. As is seen, the fainter the magnitude, the more marked this separation of the material appears.

In consideration of these peculiarities in the distribution, it is evidently not appropriate to treat the whole material as one group and use one mean value of z' for each class of apparent magnitude. Therefore the stars in each group of magnitude have been studied separately. The results are illustrated in Figs. 25 to 27. The number N in the diagrams 25 and 27 denotes the total number of stars. The number n_1 in the diagram 26 denotes the number of stars that remains when the above discussed region around the galactic centre is excluded. As is seen from the distributions, no appreciable difference is to be observed between the two results. It might therefore be concluded that the stars in the region of the galactic centre do not influence the total distribution in any special way.

The series of frequency-curves in Figs. 25 to 27 is a reflexion of the individual distribution of Figs. 19 and 20. It is of interest to observe the successive displacement of the modal values of the frequency-curves. While at the brighter magnitudes a clearly defined maximum appears near the galactic plane, and the secondary-maxima are negligible or non-existent, the fainter magnitudes have a minimum at the galactic plane, two distinct maxima on both sides symmetrically, and possibly secondary frequency-maxima. It is therefore possible to obtain in this way an indication of the position of the arms of the distribution which in this reproduction are to be identified by the maxima of the frequency-curves in each magnitude-class.

With the aid of the frequency-curves the following interpretation of the galactic distribution with regard to m has been made.

Let us consider the limits of the above-discussed group B, consisting of stars of 7^m0—11^m9. (See Fig. 24). The main maximum in the first group amounts to 10, in the second 20. Consequently we have:

$$\begin{aligned} 7^m5: \quad z_1' &= 10^{0.2m} \sin b_1 = 10; \quad z_1 = R_1 \cdot z_1', \\ 11^m5: \quad z_2' &= 10^{0.2m} \sin b_2 = 20; \quad z_2 = R_2 \cdot z_2'. \end{aligned} \quad (71)$$

Assuming:

$$\begin{aligned} z_1 &= z_2 = z, \\ R_1 z_1' &= R_2 z_2', \\ \frac{R_1}{R_2} &= \frac{z_2'}{z_1'} = \frac{2}{1}, \\ R_1 &= 2 R_2. \end{aligned} \quad (72)$$

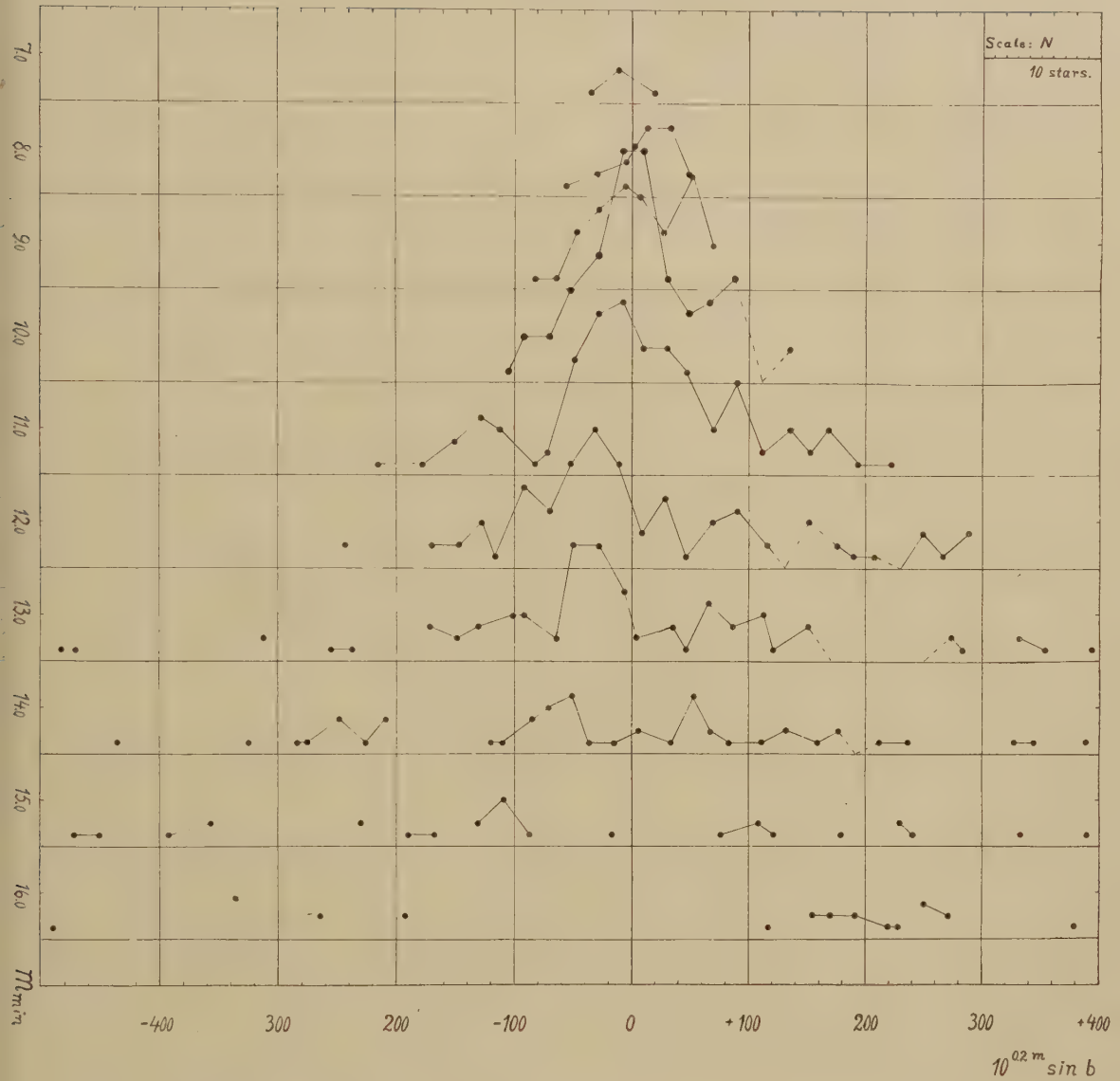


Fig. 25. Frequency-distribution in galactic z' -coordinates within each magnitude-class. In this distribution the total number N of stars is used.

We now ask to what degree this difference may depend on the correlation between M and m .

The regression-line to be used is:

$$M_1 - M_2 = r_{M,m}^2 (m_1 - m_2).$$

We have:

$$m_1 - m_2 = -4.0, \quad (73)$$

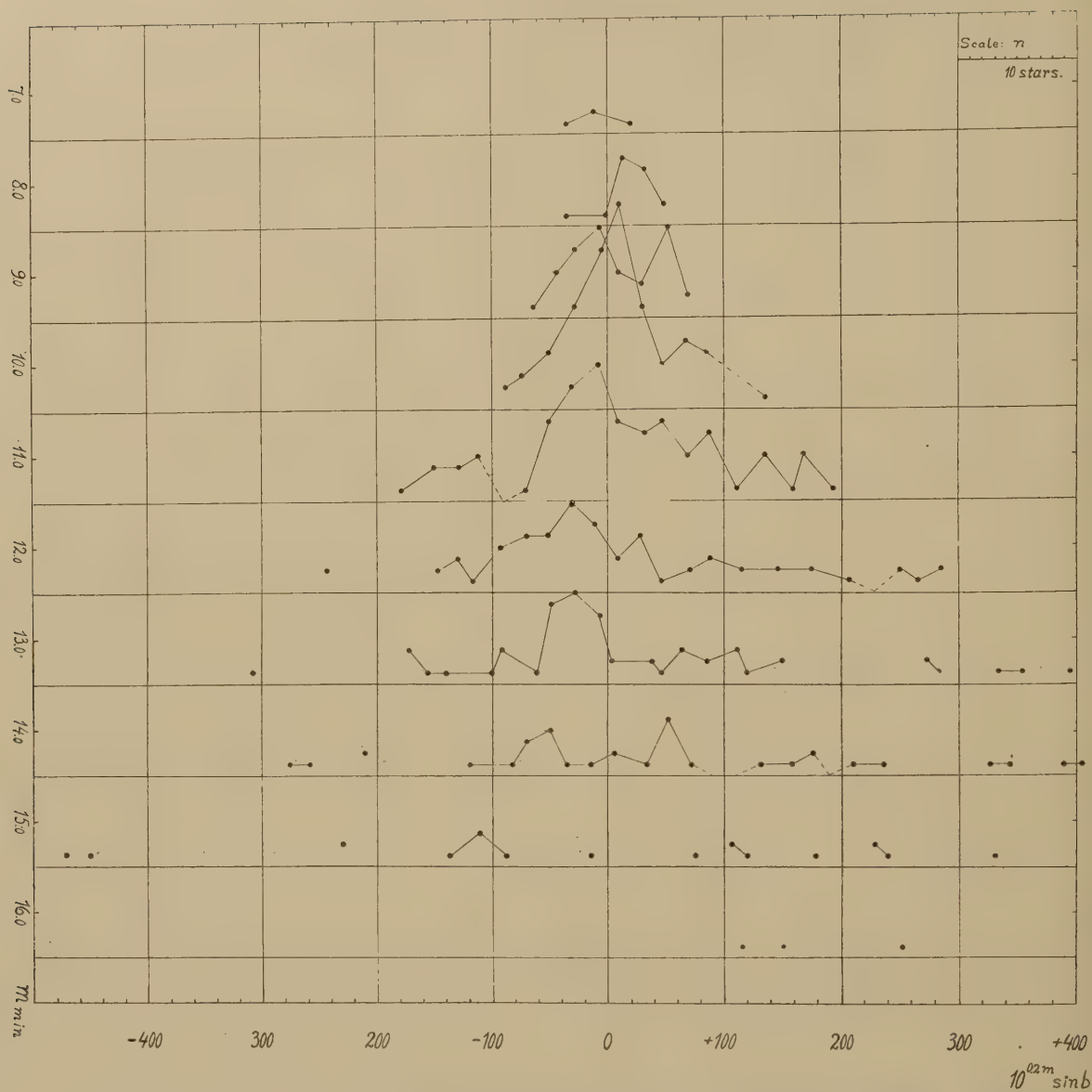


Fig. 26. Frequency-distribution in galactic z' -coordinates within each magnitude-class. In this distribution are excluded stars in the galactic centre.

Introducing the three coefficients of correlation, obtained by different methods:

1. $r_{M,m} = +0.43,$
2. $r_{M,m} = +0.62,$
3. $r_{M,m} = +0.77,$
4. $r_{M,m} = +0.82,$

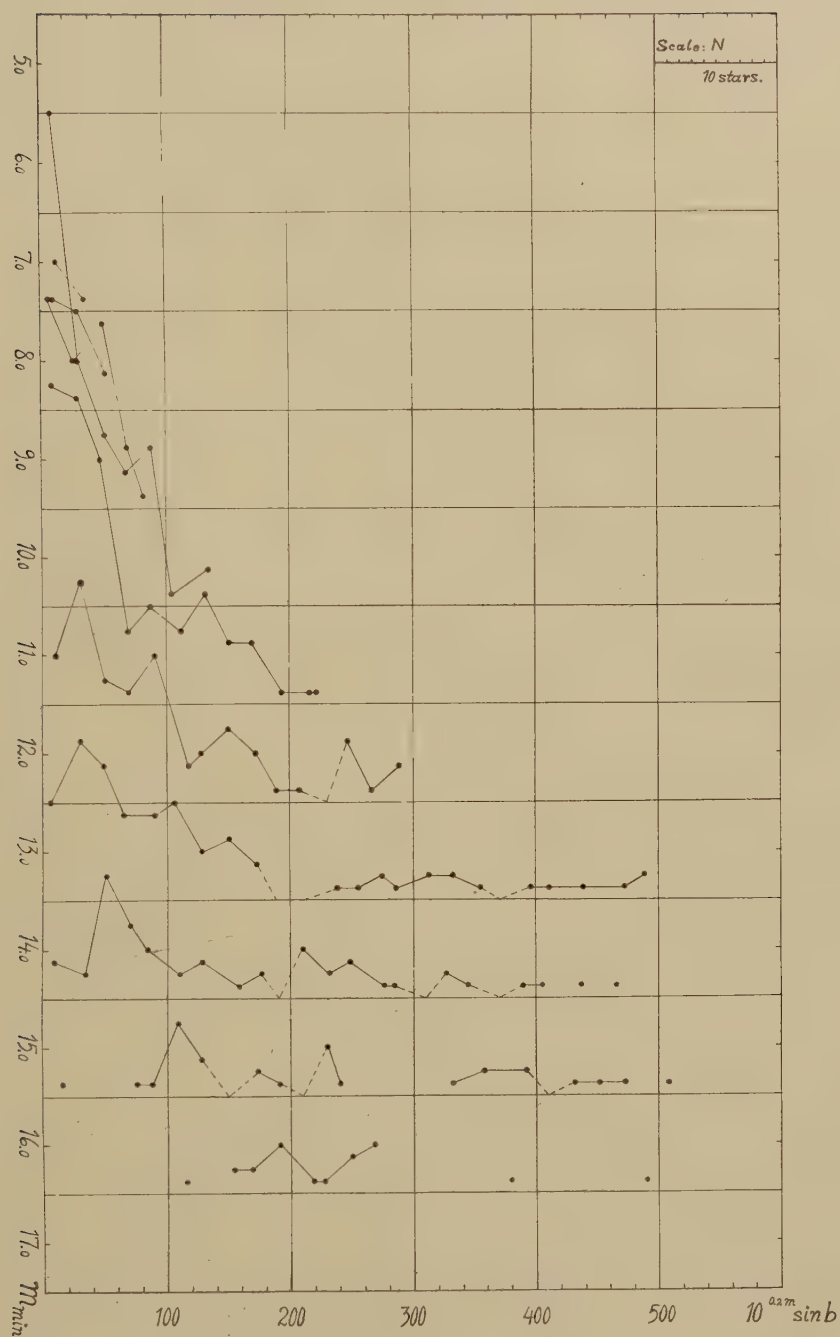


Fig. 27. Frequency-distribution in the *absolute* value of the galactic z' -coordinates, within each magnitude-class. In this distribution the total number N of the stars is used.

we obtain the following values of the absolute magnitude M_1 :

$$\begin{aligned} M_1 &= M_2 - 0.74, \\ M_1 &= M_2 - 1.54, \\ M_1 &= M_2 - 2.37, \\ M_1 &= M_2 - 2.69. \end{aligned} \tag{74}$$

$$\text{Since:} \quad R = 10^{-0.2M}, \tag{75}$$

then:

$$\begin{aligned} (1) \quad R_1 &= 10^{-0.2(M_2 - 0.74)}, \\ R_1 &= R_2 \cdot 1.41. \end{aligned} \tag{76}$$

In the same way:

$$\begin{aligned} (2) \quad R_1 &= R_2 \cdot 2.03, \\ (3) \quad R_1 &= R_2 \cdot 2.98, \\ (4) \quad R_1 &= R_2 \cdot 3.45. \end{aligned} \tag{77}$$

The condition $R_1 = 2 R_2$ is consequently satisfied by a coefficient of correlation between the values (1) and (2).

In fact, the required coefficient would be

$$r_{M,m} = +0.61. \tag{78}$$

It is evidently possible to explain the continual increase in the first frequency-maximum by attributing the effect to the correlation between apparent and absolute magnitude, since the required values are within the limits caused by the coefficients of correlation actually found. Since the first frequency-maximum might be supposed to consist of stars with small dispersion in mass and consequently also in absolute magnitude, the smaller value of the coefficient of correlation is the most probable one. Supposing this to be correct, there remains a part of the effect that might be due to absorption.

However, because of the difficulty in obtaining the data required — the exact ratio between the z' -values and the correct value of the coefficient of correlation — the above considerations, like those made in the previous part of the chapter, must be only approximate.

However, it seems probable that the increase in z' might in this instance be regarded as mainly due to the correlation between M and m .

We are now going to consider the first frequency-maximum of z' at $14^m.5$, which represents the limiting magnitude for the present investigations.

The branches of the curves are here separated from the galactic plane. The mode-value lies at $z' = 50$.

As before:

$$M_1 - M_3 = r_{M,m}^2 (m_1 - m_3).$$

Hence:

$$\begin{aligned} (1) \quad & M_1 - M_3 = -1.29, \\ (2) \quad & M_1 - M_3 = -2.69, \\ (3) \quad & M_1 - M_3 = -4.15, \\ (4) \quad & M_1 - M_3 = -4.71. \end{aligned} \tag{79}$$

Then:

$$\begin{aligned} (1) \quad & R_1 = 1.82 R_3, \\ (2) \quad & R_1 = 3.45 R_3, \\ (3) \quad & R_1 = 6.76 R_3, \\ (4) \quad & R_1 = 8.74 R_3. \end{aligned} \tag{80}$$

The required ratio is 5.

This is satisfied for a value of $r_{M,m}$ between (2) and (3).

Again, supposing that the stars belonging to the frequency-maximum near the galactic plane define the absolutely bright group of the irregular variables referred to above, the same coefficient of correlation would have to be valid for the group as a whole, provided the correlation actually is linear.

If we assume the correlation to be represented by the smallest coefficient $r_{M,m} = +0.43$, the effect caused by the correlation can be calculated. Now we have observed z' equal to 50. The value arrived at when no correlation and no absorption is present would be $z = 10$.

The value obtained when the correlation is taken into account is $z_2' = R z_1' = 10 \times 1.82 = 18$. Hence the effect in the magnitude from an absorption is easily deduced.

The true magnitude m' would give $z' = 18$.

The actually observed magnitude m'' gives $z' = 50$. Then we have:

$$10^{0.2m'} \sin b = 18 \text{ and } 10^{0.2m''} \sin b = 50.$$

$$\text{So:} \quad 10^{0.2(m'' - m')} = \frac{50}{18} = 2.778,$$

$$\text{and:} \quad m'' - m' = 2^m 2. \tag{81}$$

This is the correction to be applied to $m = 14.5$, provided the stated effect in z' is due to absorption. This correction corresponds to a general coefficient of absorption that is equal to or rather less than one magnitude per kiloparsec. We can write, denoting α as the coefficient of absorption,

$$\alpha \leq 1^m \text{ per 1000 parsecs.}$$

In the above discussion I have not taken into consideration a possibly existing variation with distance (or with apparent magnitude) in the dispersion of the absolute magnitudes.

In fact the general expression of the quantity R has the form ⁹:

$$R_m = e^{-bm} \cdot \bar{r}$$

or:

$$R_m = e^{-bM_0 - b(1-\lambda_1)(m-m_0) + \frac{1}{2}b^2\sigma_M^2\lambda_1}, \quad (82)$$

where we observe that

$$\lambda_1 = 1 - r_{M,m}^2.$$

From expression (82) it appears that a possible change of σ_M with e. g. the distance of the stars can considerably affect the calculated scattering of the distribution of the z -coordinates.

The supposition has been suggested above that the secondary branches of the general frequency-curve consist of stars of less absolute magnitude than those situated nearer the galactic plane. We will now try to determine the difference in magnitudes that would give rise to the observed effect.

Considering the frequency-curve of magnitude 12.5, where the several maxima are clearly identifiable, we read the following values:

1:st max.	2:nd max.	3:rd max.
$z' = 30$	$z' = 90$	$z' = 250$.

Further:

$$z = R 10^{0.2m} \sin b. \quad (83)$$

Then:

$$z_1 = R' z'_1, \quad z_2 = R'' z'_2, \quad z_3 = R''' z'_3. \quad (84)$$

Assuming the mean value of z to be the same we obtain:

$$R' = 3 R''. \quad (85)$$

Now: $M = -5 \log R.$

So: $M' = -5 \log R' = -5 \log 3 R'',$

and: $M'' = -5 \log R'', \quad (86)$

$$M' = -5 (\log 3 + \log R'') = -5 \log 3 - 5 \log R''.$$

⁹ Cf. CHARLIER: Studies in Stellar Statistics No. 1, Lund Medd Ser II No. 8 (1912).

Consequently:

$$\begin{aligned} M' &= M'' - 5 \log 3, \\ \text{then: } M' &= M'' - 2.39. \end{aligned} \tag{87}$$

Introducing the third maximum we obtain:

$$R' = \frac{25}{3} R'' = 8.333 R''.$$

Hence:

$$M' = M''' - 4.60. \tag{88}$$

These values give a difference in absolute magnitude between the three different arms of the frequency-curve of 2^m3 , the brighter stars lying nearest to the galactic plane.

It is of interest to observe that this difference coincides fairly well with the difference between the layers in the period-apparent-magnitude curve constructed by GYLLENBERG for the long-period variables.¹⁰

There seems consequently to be a certain probability of the existence of several preferential values of absolute magnitudes among variable stars, and that there appears a difference between successive values of approximately 2^m3 .

As has been pointed out, the large coefficient of correlation between M and m is in itself a proof that the irregular variables probably consist of members of fairly different absolute magnitudes.

The existence of irregular variables of faint absolute magnitudes, dwarf variables, has been indicated with a certain degree of probability by S. GAPOSCHKIN.¹¹ As the author himself has pointed out, the stars in question might possibly be up to 3 magnitudes brighter than the values obtained by him, owing to the difficulty of determining the distance modulus to the star-cloud. Then the dwarf-nature of at least the first three stars of his list would be problematic. The last three would undoubtedly be very faint stars.

The supposition of the differences in absolute magnitudes for stars with different galactic latitudes has also been proved by the determinations of the absolute magnitudes from the proper motions. The stars with small galactic latitude were found to be considerably brighter than those outside the galactic plane.

¹⁰ Lund Medd Ser II No. 54 p. 14 (1930).

¹¹ Harvard Tercentenary Paper No. 27 (1938).

CHAPTER 7

The radial velocities

1. The sources.

Radial velocities are known for only a comparatively small number of the irregular variables. Data have been collected from the General catalogue of radial velocities of Lund Observatory compiled by Dr. H. NORDSTRÖM,¹ and augmented from the recent catalogue of radial velocities for *R* and *N* stars by R. S. SANFORD.² This catalogue has contributed 50 new radial velocities, and newly determined values for a number of older ones. The present material contains 84 radial velocities.

2. The frequency-distribution.

The frequency-distribution of the radial velocities corrected for the solar motion is represented in Table 12 and Figs. 28 and 29. The latter gives the distribution of the radial velocities when no account is taken of the sign of these quantities.

The radial velocities generally show fairly small values, the greatest number (58 stars) lying below 30 km/sec. Two large values: X Her with $W = -91.1 \pm 0.7$ km/sec and V Ari with -183 km/sec (uncorr.) are recorded.

In the following calculations no account has been taken of V Ari. X Her, lying on the limit of three times the dispersion, is not omitted. Alternative calculations have shown unappreciable differences with the velocity of this star omitted or not.

Thus the following parameters have been obtained:

$$\begin{aligned}\overline{W} &= +0.4 \pm 2.7 \text{ km/sec} \\ |\overline{W}| &= 19.4 \pm 1.5 \text{ km/sec} \\ \sigma &= 24.3 \pm 1.9 \text{ km/sec.}\end{aligned}\tag{89}$$

For a normal distribution the following relation is valid: $\sigma = \sqrt{\frac{\pi}{2}} \vartheta$. Therefore, if we find that the figures obtained satisfy this equation, as also was found in the

¹ Lund Medd Ser II No. 79 (1936).

² ApJ 82 No. 2 p. 202 (1935).

³ MtW Contr 525 (1935).

TABLE 12.

Distribution of the radial velocities of the irregular variables.

a)			b)			c)		
W km/sec	N_1	N_2	$ W $ km/sec	N_1	N_2	S_p	N	$ \overline{W} $ km/sec
+ 55 to + 59		2	0 to 4	11		B	1	54.
50 » 54	2		5 » 9	15	26	A	1	10.
45 » 49	2	3	10 » 14	15		F	—	—
40 » 44	1		15 » 19	7	22	G	2	16.0
35 » 39	2	7	20 » 24	5		K	4	24.2
30 » 34	5		25 » 29	5	10	M	16	20.4
25 » 29	3	6	30 » 34	10		N	54	19.5
20 » 24	3		35 » 39	5	15	R	3	73.3
15 » 19	1	10	40 » 44	3		S	1	16.
10 » 14	9		45 » 49	2	5	Pec	2	23.
+ 5 » + 9	9	14	50 » 54	2	4			
+ 0 » + 4	5		55 » 59	2				
— 0 » — 4	6	12	60 » 64	—				
— 5 » — 9	6		65 » 69	—				
10 » 14	6	12	70 » 74	1	1			
15 » 19	6		75 » 79	—				
20 » 24	2	4	192	1	1			
25 » 29	2		All	84	84			
30 » 34	5	8						
35 » 39	3							
40 » 44	2	2						
45 » 49	—							
50 » 54		2						
— 55 » — 59	2							
— 73	1	1						
— 192	1	1						
All	84	84						

present case, we conclude that the distribution is a normal one. Consequently the radial velocities do not so far indicate a separation of the present material into two types. Such a separation may still exist, but then one type must be less dominant than the other.

3. Determination of the coordinates of apex and the Sun's velocity.

The coordinates of the apex and the Sun's velocity relative to this group of stars have been determined from the radial velocities with methods extensively described in several standard works. So we shall not enter upon these methods here. For a more detailed study of methods reference may be made to works of C. V. L. CHARLIER, W. GYLLENBERG, G. MALMQUIST, H. NORDSTRÖM and others.

The equation of conditions to be considered has the following form:

$$W = \gamma_{13} U'' + \gamma_{23} V'' + \gamma_{33} W'' + K. \quad (90)$$

Owing to the small material, the direction-cosines γ have been computed individually.

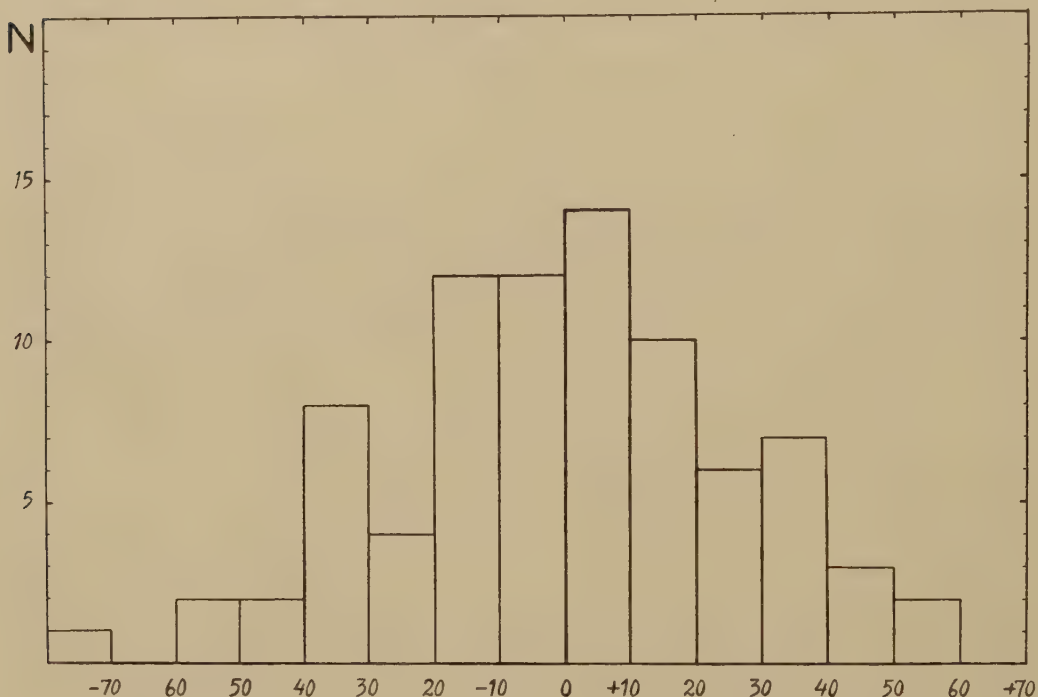


Fig. 28. Distribution of the radial velocities of the irregular variables.

It has been found from investigations that irregular variables for which the radial velocities are known are fairly uniformly distributed over the two galactic hemispheres. Considering this fact, an abbreviated solution of the normal equations for obtaining the coordinates of the apex and the Sun's velocity might be justified. An even distribution of the stars over the sky is numerically expressed by the well-known fact that the following sums, entering the normal equations, are equal to zero:

$$\Sigma \gamma_{13} \gamma_{23}, \Sigma \gamma_{13} \gamma_{33}, \Sigma \gamma_{23} \gamma_{33}. \quad (91)$$

However, because of the scantiness of the material it was found more convenient to perform a strict computation, viz. to take into account the fact that the above-mentioned sums are *different* from zero.

The following values have then been obtained:

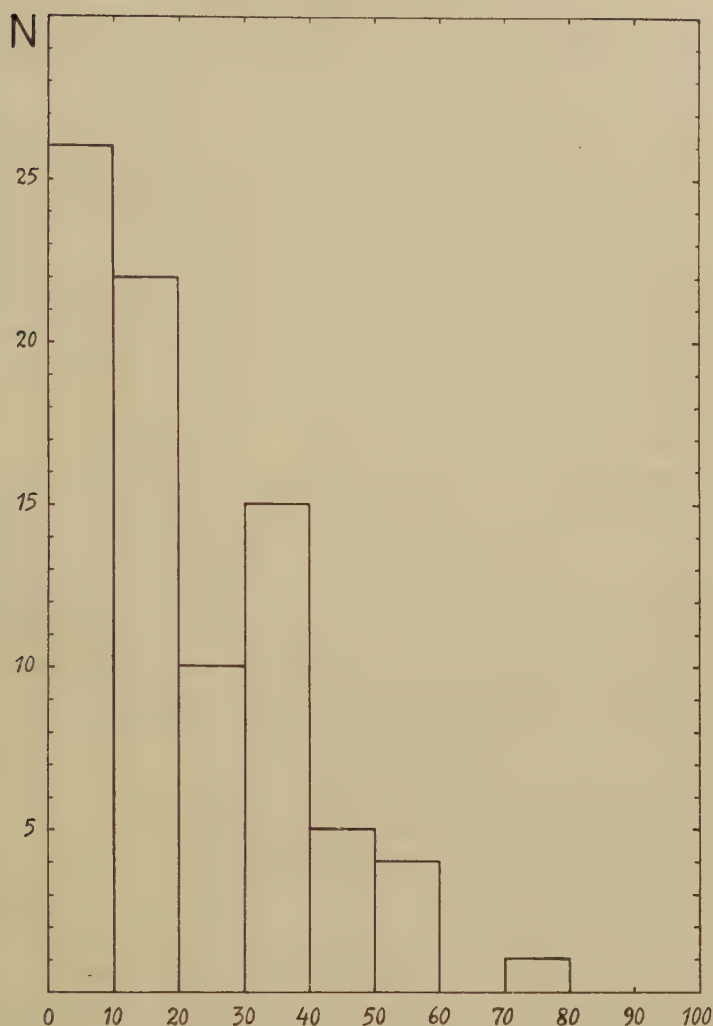


Fig. 29. Distribution of the radial velocities of the irregular variables, when no account is taken of the sign.

(I)	(II)	
$A = 270^\circ$	$A = 310^\circ$	
$D = +37^\circ$	$D = +32^\circ$	
$S = -21.3 \pm 5.3 \text{ km/sec}$	$S = -19.9 \pm 5.3 \text{ km/sec.}$	(92)

In case I the star V Ari is excluded.

For the mean errors in the coordinates of the apex, cf. p. 110.

4. Comparison with other classes of stars.

It may be of interest to compare the derived solar velocities with results for other groups of stars.

In his paper on radial velocities H. NORDSTRÖM⁴ has determined the Sun's velocity and the coordinates of the apex for different groups of stars. The K and M stars are of special interest in this connexion.

The agreement is remarkable between our results obtained for the irregular variables and those for the M-stars. This agreement might indicate that all M-stars, thus also those without emission-lines, are *variable* stars and of the class known as irregular. New variables of this kind are constantly discovered. A special group of irregular variables form those of very small amplitudes; their change in brightness is only measurable by the photo-electric cell. The registered number of this kind of variables is rapidly increasing. It is yet too early to decide if these stars are of the same kind as those having large amplitudes. But there are, however, several facts which point in this direction.⁵

The agreement between the solar velocity of the irregular variables and that of the M-stars in general is a fact to take into consideration. The long-period variables of spectral class M have a considerably larger solar motion, which amounts to 50.7 km/sec.⁶ The Cepheid variables, on the other hand, have a smaller velocity. The value found by GYLLENBERG in the mentioned investigation is 11.2 km/sec. NORDSTRÖM has found for this velocity 16.4 km/sec on the basis of a larger material.

Looking at the series of tabulated values of solar motions, it is of interest to note that the irregular variables satisfy a position between the Me-variables and the Cepheids.

⁴ Lund Medd Ser II No. 79 (1936), p. 90, 96, 98.

⁵ PARENAGO RAJ XI.1 (1934).

⁶ GYLLENBERG: Lund Medd Ser II No. 54 p. 15 (1930).

CHAPTER 8

The proper motions

1. Determination of the proper motions.

a. Positions.

In the present catalogue are given 70 newly derived proper motions, and 112 which have been revised from the author's earlier determination of positions and proper motions.¹ They have been computed after addition of a newly determined position, obtained by myself with the meridian circle of the Lund Observatory.²

For a number of stars the proper motion has been calculated when other modern observations could be added. In this way several new proper motions were determined. In some other cases the old proper motions were used uncorrected.

In order to prepare the meridian-observations the available material of irregular variables was investigated as to the possibilities for proper motion determinations. The compiled stars were entered in the program of observation. It is of course of great value to secure positions of variable stars which never before had their accurate position determined. In the list of positions a few stars of this kind are present. Since the difficulties in getting meridian-positions for faint variables are often considerable, the observations have been extended over several years.

The positions have been discussed in the catalogues referred to. But for the sake of uniformity a short survey of the methods and results is given here.

b. The fundamental system of the positions.

In my first catalogue³ the positions were referred to the system NFK. Later on, the system FK3 has been adopted as fundamental system. The corrections to be applied to NFK in order to pass over from that to FK3 are for some of the fundamental stars considerable. The individual differences in the two reference-systems will naturally influence the result from the series of observations when referred to the fundamental system. In the earlier series of observations it was sometimes noted that a certain fundamental star differed to a considerable degree

¹ Lund Medd Ser II No. 66 (1932).

² Loc. cit.; LOC No. 12 (1937).

³ Loc. cit.

from the average reference-curve, and did so during several nights of observation. The differences between the observed positions of the stars and the system FK3 will as a rule show a smaller distribution around a mean curve than did the same differences when the system NFK was used.

In this connexion another fact may be mentioned. It has been stated that we have adopted the fundamental system FK3 or NFK. However, a number of stars of *Berliner Jahrbuch* are not provided with ten-day ephemerides. Since these stars are frequently used in the program, their positions have been calculated from the ephemerides in *Almanaque Náutico de San Fernando*. These ephemerides refer to the system of W. S. EICHELBERGER (since the year 1928), and the stars are reduced to FK3 by means of their mean places.

c. The derivation of the proper motions.

The proper motions were all computed by means of the method of least squares from as many as possible of the existing positions. For the observations before 1900 the positions were collected from »Geschichte des Fixsternhimmels» which gives the positions reduced to the equinox 1875.0 by means of STRUVE's precession-constant. From 1900 to 1925 the *Bergedorf Index-Catalogue* gives references. After 1925 the individual catalogues have been consulted in each case. All the stars have been reduced to 1875.0 by means of STRUVE's precession-constant.

On the average the proper motions computed in this paper were based on 8.2 positions. As a rule 3 positions were accepted as a minimum number for a computation.

Together with the meridian-catalogues the photographic catalogues of *Carte de Ciel* were also searched. Since these catalogues as a rule only give rectangular coordinates, the work of identifying a star is often considerable. Exceptions are the *Helsingfors* and earlier *Catania Catalogues*, which give accurate equatorial coordinates for 1900.0, and *Potsdam* with approximate equatorial coordinates. A difficulty with the photographic catalogues is the establishing of their fundamental system. In fact, it is not possible to define a fundamental system for them. Therefore they cannot be corrected for systematic differences. This is of course a drawback, because of the comparatively great weight that is attached to modern photographic catalogues. From the investigations performed at *Helsingfors* concerning the systematic corrections of the photographic catalogue⁴ it appears that these quantities are by no means negligible. On the contrary, they are surprisingly large. Therefore the systematic corrections cannot be put equal to zero, as in fact has been done in the present work. To decrease the influence of the uncorrected positions, the weights have to be given a lower value than would be normally assumed.

⁴ H. O. GRÖNSTRAND: Bestimmung der Systematischen Korrekturen der Sternörter im photographischen Zonenkataloge der Sternwarte Helsingfors (1937).

For the systematic corrections the following procedure has been adopted.

It was decided to refer the proper motions to the system of Boss, defined by Boss's Preliminary General Catalogue (PGC), since systematic corrections to this system are available for a considerable number of catalogues. They are given in the Preliminary General Catalogue of Boss⁵ and the pamphlet »Systematic corrections and weights of catalogues» of Roy.

Other catalogues, not included in the system of PGC, are corrected to the system NFK⁶ by A. AUWERS. To these corrections should be added the correction (Boss-NFK) for the certain equinox. This correction has been computed by the present author by means of tables.⁷

But there still remains a number of catalogues for which no systematic corrections have been derived. Some are older catalogues, which are not likely to be made frequent use of, but most of them are modern catalogues. In these cases the catalogue has itself been consulted for references to the fundamental system used. The older catalogues do not contain any corrections to a definite system. Hence e. g. star-catalogues not occurring in the above mentioned lists, referring to the system FK, have been utilized after the coordinates have been corrected by the quantities

$$(\text{NFK} - \text{FK}) + (\text{Boss} - \text{NFK}).$$

These corrections have been calculated by the present author by means of the tables by AUWERS in AN 3927.⁸ For modern catalogues the problem is as a rule easier, since the observers always refer their observations to a definite fundamental system. The necessary corrections to the system of Boss can therefore in most cases easily be applied.

The tables of Boss or Roy, giving the systematic corrections of a large number of catalogues, also contain the corresponding weights.

For other catalogues the same system of weights has been adopted. The weights have been estimated from the mean errors of the catalogue. In the present computations the upper limit of weights has been put = 3.0. There is a risk that too large weights in certain observations might result in a proper motion that is to a great extent the consequence of the calculations.

The proper motions were computed by the central-epoch method explained in my earlier paper.⁹

⁵ Since the completion of the present work the new Boss Catalogue has appeared, giving newly computed systematic corrections and weights. However, the present investigations being by this time completed, a reference to the new Boss system could unfortunately not be made.

⁶ AN Ergänzungsheft 7 (1904).

⁷ Astronomical Papers of the American Ephemeris and Nautical Almanac, Vol X, Part I, p. 150 (1925).

⁸ AN 164.225 (1904).

⁹ See the more extensive accounts in Lund Medd Ser II No. 66 (1932).

2. The proper-motion material used in the present investigations.

The present material contains proper motions for 228 irregular variables, collected from the following sources:

	Number of stars
1. F. Palmér	123
2. R. Wilson	46
3. W. Gyllenberg	17
4. Boss PGC	14
5. P. P. Parenago	11
6. C. Hins	5
7. Various authorities	13

The proper-motion material is referred to a uniform system. In the author's computations of the proper motions the positions have been referred to the system of Boss. The proper motions have been derived from least-square solutions. The precession-constant used is that of NEWCOMB. The proper motions compiled from the other catalogues have all been determined under mainly the same conditions. Of course this fact simplifies further work, since no corrections to a uniform system need be applied.

In the course of the work, however, it was found to possess a certain interest to introduce the system FK3 into the proper motions. Some of the computations have been carried out parallel in the two systems, since it was thought of interest to compare the results obtained. Since most tables for systematic corrections of the different star-catalogues refer to the system of Boss, the system FK3 was not used in the original computations of the proper motions. The work of transferring the individual catalogues to FK3 would at present be complicated, whereas the application of such corrections to the final proper motions is a simple procedure. The transformation was made by means of the tables contained in several papers from the Astronomisches Recheninstitut.¹⁰

It is known that the proper motions of Boss PGC need a correction in declination. In applying the corrections transforming the proper motions to FK3 it was found that a rather large positive correction to the Boss proper motion in declination is required. In right-ascension, however, the corrections are for the most part negative. The corrections needed are sometimes of the same order of magnitude as the proper motions themselves.

¹⁰ AN 255, No. 6113—4, Mitteilungen des Astronomischen Recheninstituts Berlin-Dahlem, Bd 3, No. 21 Kiel (1935), by H. NOWACKI: »Vergleich mit den Fundamentalkatalogen von A. AUWERS, L. BOSS und W. S. EICHELBERGER für 1925.»

3. Statistical investigation of the proper motions.

The proper motions of the irregular variables are in most cases very small quantities. The following three stars have been excluded, since their proper motions are considerably larger than those of the material in general:

α Oph	$\mu = ".294$
ϱ Per	.173
VY Cas	.165

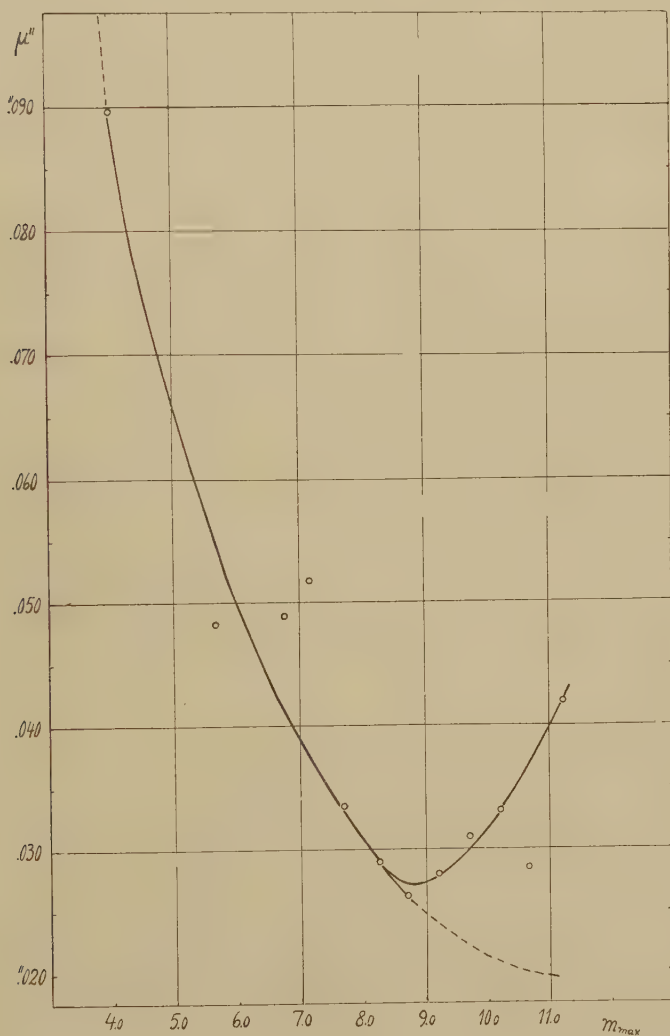


Fig. 30. Correlation between the proper motion and the apparent maximum magnitude [No. 225].

The material used above is extended over the apparent photographic magnitudes $m_{\text{max}} < 11^{\text{m}}.5$. The proper motions of stars with faint magnitudes are expected to be of less accuracy than those of the bright stars, owing to the difficulties connected with the observations of faint stars. From the dependence of the total proper motion μ on apparent magnitudes, illustrated in Fig. 30, it is possible to determine the limit of the utility of the material. Since the apparent magnitudes are approximately a measure of the distances of the stars, the proper motions of the faint stars tend continuously towards zero. But this is not the case in Fig. 30. Until $m_{\text{max}} < 9^{\text{m}}.5$, the proper motions rapidly decrease, but after this magnitude the curve again tends upwards. Hence it can be concluded that the proper motions of stars fainter than $m_{\text{max}} = 9^{\text{m}}.5$ are affected by considerable errors and ought therefore to be omitted.

However, it may be pointed out that errors may also occur in the apparent magnitudes. The latter quantities are difficult to establish for these stars. Further an absorption of light in space which has been discussed elsewhere in this paper may cause a systematically decreasing brightness of the stars. This would, of course, affect the mean values of the proper motions.

Probably both effects are present. Since they have not been determinable in the present investigations, the material has been treated alternately as a whole and with the magnitudes $> 9^{\text{m}}.5$ excluded.

CHAPTER 9

Determinations from the proper motions of the coordinates of apex

The method of solution.

The method here adopted to determine the coordinates of apex from the proper motions is well-known. For the details I refer to previous papers of CHARLIER and his collaborators.

The numerical solutions.

The determinations of the coordinates of apex were carried out with non-reduced proper motions and with proper motions reduced to a unit-distance by means of the photometrical factor $10^{0.2m}$.

The direction cosines γ_{ij} have been calculated individually for the stars.

In the normal equations the sums of the double product-terms were neglected, since the irregular variables have been found to be practically symmetrically distributed over both hemispheres. As a test the strict solution was applied in the case of the non-reduced proper motions, using the whole material. The difference from the results of the short solution is inconsiderable.

In the following the abbreviated solution [$\Sigma \gamma_{ij} = 0$] has therefore been used to determine the coordinates of apex and the absolute magnitude.

In this way the following values of the coordinates of apex, in Table 18, have been obtained.

TABLE 18.
Coordinates of apex.

Apex	From radial velocities	From proper motions					
		Non-reduced proper motions					Red. p. m. ($m_{\max}$)
		Short solution	Strict solution	Short solution $m_{\max} = 6.5-10.4$	$m_{\max} = 5.0-9.4$		
					System: Boss	System: FK3	
A	270°	303°	288°	305°	281°	299°	332°
D	+ 37	+ 54	+ 53	+ 43	+ 44	+ 22	+ 63

It is seen from the table that the values of the coordinates of apex, obtained from the proper motions, to a certain extent differ from those determined by means of the radial velocities.

The mean errors in both coordinates of apex have been derived from the different solutions. They have been found to agree approximately. Therefore the following average value of the mean errors has been adopted.

We have:

$$\varepsilon(A) \simeq \varepsilon(D) \simeq \pm 18^\circ.$$

The last two columns represent the best of the values obtained from the proper-motion material to hand, owing to the chosen limits of apparent magnitude. Investigations on the proper motions have shown that the stars of magnitudes fainter than $9^m.5$ are affected by a fairly large uncertainty. Therefore the best results would be expected from a material where these proper motions are omitted. In fact the coordinates thus obtained approximate to those obtained from the radial velocities.

Further it is of interest to observe the effect of the change from one fundamental system to the other, the importance of which has been more extensively considered elsewhere in this paper. Here it might be pointed out that the proper motions referred to FK3 give a declination that is displaced in a direction towards that obtained from the radial velocities, and that found for stars in general.

A similar effect in declination has been found by R. E. WILSON and H. RAYMOND in their recently published extensive investigation on the new BOSS-Catalogue.¹ The effect of the fundamental system on the coordinates of apex, determined from the proper motions, has been discussed with reference to the systems GC, PGC and FK3. The difference between the value of the right-ascension in the three systems is inconsiderable, whereas the declination turns out about 10° smaller in FK3 than in GC and PGC. This result is in close accordance with that obtained in the present investigation, with due regard taken to the mean errors.

¹ AJ XLVII.6 No. 1084 (1938).

CHAPTER 10

The absolute magnitudes

The absolute magnitudes as derived from the proper motions

When the mean apparent magnitude m for a group of stars is known, the mean absolute magnitude M_m for this group is obtained from the relation:

$$M_m = m - \overline{5 \log r_m}, \quad (93)$$

where the unit of distance is assumed to be 1 decaparsec (= 10 parsecs). The distance modulus, $\overline{5 \log r_m}$, is derived from the mean inverse value of the distance — $\overline{1/r}$ or ϑ_1 , which have been obtained from a treatment of the proper motions described above.

In deriving the mean distance $\bar{r}$ from ϑ_1 , it is not sufficient merely to determine the inverse value of this quantity. The value thus obtained will need a correction, depending on the dispersion in absolute magnitude. Then we may write:

$$\bar{r} = k \cdot \frac{1}{1/r}. \quad (94)$$

The factor k here represents the correcting factor. A term of correction, caused by the mentioned dispersion, will also enter the formula (93).

This matter has been discussed in various papers by CHARLIER,¹ who first gave an extensive derivation of the relation between the quantities $\bar{r}$ and $\overline{1/r}$.

Since the relations mentioned are of importance in the present discussion as to the connexion between the mean absolute magnitudes and the mean distances, a short survey shall be given of the methods to obtain the constant k and the distance modulus.

Starting from the definition of a statistical mean value:

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} x f(x) dx}{\int_{-\infty}^{+\infty} f(x) dx}; \quad (95)$$

¹ Lund Medd Ser II No. 9 (1913).

and observing that:

$$a(m) = \omega \int_0^{\infty} D(r) r^2 \varphi(M) dr, \quad (96)$$

we obtain different expressions, depending on the assumptions concerning the density function $D(r)$.

The frequency-function $\varphi(M)$ of the absolute magnitudes can in the first approximation be assumed to be represented by a Gaussian function:

$$\varphi(M) = \frac{\text{const}}{\sigma_M \sqrt{2\pi}} e^{-\frac{(M - M_0)^2}{2\sigma_M^2}}, \quad (97)$$

where σ_M is the dispersion in M .

1. $D(r)$ is regarded as constant.

We obtain:

$$\bar{r} a(m) = \text{const} \int_{-\infty}^{+\infty} e^{-4by} \cdot e^{-\frac{(m+y-M_0)^2}{2\sigma_M^2}} dy, \quad (98)$$

$$\overline{1/r} a(m) = \text{const} \int_{-\infty}^{+\infty} e^{-2by} \cdot e^{-\frac{(m+y-M_0)^2}{2\sigma_M^2}} dy. \quad (99)$$

Here we have introduced y as an independent variate, defined in the following way:

$$y = -5 \log r, \text{ or: } r = e^{-by}, \quad (100)$$

where:

$$b = \frac{1}{5 \text{ Mod}} = 0.4605.$$

Evaluating the integrals above, we obtain the following expressions for $\bar{r}$ and $\overline{1/r}$:

$$\bar{r} = e^{\frac{7}{2} b^2 \sigma_M^2 - b(M_0 - m)}, \quad (101)$$

$$\overline{1/r} = e^{-\frac{5}{2} b^2 \sigma_M^2 + b(M_0 - m)}. \quad (102)$$

From e.g. the parallactic motion of the stars compared with their radial velocities we directly obtain the numerical value of the inverse distances.

Inverting the value of $\frac{1}{1/r}$, we evidently do not arrive at the above expression

for $\bar{r}$. We have in fact to multiply $\frac{1}{1/r}$ by $e^{b^2 \sigma_M^2}$.

Thus we have arrived at the well known relation:

$$\bar{r} = \frac{1}{1/r} e^{b^2 \sigma^2 M}. \quad (103)$$

Since the factor $e^{b^2 \sigma^2 M}$ is always > 1 , the mean value of r is larger than would be found from inverting $\overline{1/r}$. If the absolute magnitude were constant, the two values would coincide, since the factor $e^{b^2 \sigma^2 M}$ is equal to 1, when the dispersion is zero.

In order to derive the absolute magnitude M_m corresponding to a certain apparent magnitude m , we have to determine the distance modulus, $\overline{5 \log r}$. This mean value is defined by the relation: $m - M_m = \overline{5 \log r_m}$.

The mean value of $(5 \log r)$ is statistically found from the following expression:

$$\overline{5 \log r_m} = \frac{\int_0^\infty D(r) r^2 5 \log r \varphi(M) dr}{\int_0^\infty D(r) r^2 \varphi(M) dr}. \quad (104)$$

For the sake of convenience the independent variate M is substituted for r . We obtain:

$$\overline{5 \log r_m} = \frac{\int_{-\infty}^{+\infty} e^{3b(m-M)} (m - M) \varphi(M) dM}{\int_{-\infty}^{+\infty} e^{3b(m-M)} \varphi(M) dM}. \quad (105)$$

Introducing the expression (97) for $\varphi(M)$ and performing the integration, we find:

$$\overline{5 \log r_m} = m - M_0 + 3b \sigma_M^2. \quad (106)$$

By means of the above relation (93) we now obtain the mean absolute magnitude for a group of stars M_0 , if the mean absolute magnitude corresponding to an apparent magnitude m is known.

Thus we arrive at the well-known expression:

$$M_0 = M_m + 3b \sigma_M^2. \quad (107)$$

The values of ϑ_1 , as obtained from the proper motions, give us a means to compute the mean distance modulus $\overline{5 \log r}$.

Starting from the value of $\overline{1/r}$ as given above (102) and observing that:

$$\frac{1}{2} b \epsilon \log \vartheta_1 = \overline{5 \log r},$$

we get:

$$\begin{cases} 5 \log \vartheta_1 = -\frac{5}{2} b \sigma_M^2 + M_0 - m, \\ \overline{5 \log r} = m - M_0 + 3b \sigma_M^2. \end{cases}$$

Then:

$$\overline{5 \log r} + 5 \log \vartheta_1 = \frac{5}{2} b \sigma_M^2. \quad (108)$$

When ϑ_1 is known we calculate the distance modulus in the unit of 10 parsecs. Combining (108) with the relation (93) we are able to determine M_m , when m is given.

$$M_m = m + 5 \log \vartheta_1 - \frac{5}{2} b \sigma_M^2. \quad (109)$$

The term $\frac{5}{2} b \sigma_M^2$ is the correction due to the calculation of M from the statistical mean value of $1/r$, and depends on the dispersion in absolute magnitude.

2. The density-function $D(r)$ is represented by the following expression:

$$D(r) = \gamma \cdot r^{-\kappa},$$

where γ and κ are constants.

With this assumption we obtain the same relation between $\bar{r}$ and $\overline{1/r}$ as above in case 1).

In comparison with the former case the correction to the distance modulus and absolute magnitude on account of the dispersion in absolute magnitude will also remain unchanged.

In order to transfer M_m to the mean absolute magnitude M_0 , however, an additional correction will be required, depending on the constant κ .

We find:

$$M_0 = M_m + b \sigma_M^2 (3 - \kappa). \quad (110)$$

3. Log r normally distributed.

Proceeding in a way similar to those cited above we obtain the following expression for the mean value of r :

$$\bar{r} = \frac{1}{1/r} e^{b^2 \sigma_M^2 \lambda_1}. \quad (111)$$

The factor λ_1 is defined by:

$$\lambda_1 = \frac{\sigma_y^2}{\sigma_m^2} = 1 - r_{M,m}^2, \quad (112)$$

where: σ_y is the dispersion in y , when $y = -5 \log r$,

σ_m is the dispersion in apparent magnitude,

$r_{M,m}$ is the coefficient of correlation between M and m .

For the distance modulus the following relation is obtained:

$$\overline{5 \log r} + 5 \log \vartheta_1 = \frac{5}{2} b \sigma_M^2 \lambda_1. \quad (113)$$

Then:

$$M_m = m + 5 \log \vartheta_1 - \frac{5}{2} b \sigma_M^2 \lambda_1. \quad (114)$$

The correction required to transfer M_m to M_0 is given by the expression:

$$M_0 = M_m - (m - m_0)(1 - \lambda_1) + 2 b \sigma_M^2 \lambda_1. \quad (115)$$

We observe that we have:

$$\lambda_1 = 1 - r_{M,m}^2.$$

Hence λ_1 is determined by the correlation-coefficient $r_{M,m}$, the numerical value of which in most cases is very difficult to obtain.

In fact, when all stars are treated together without spectral distinction, the above coefficient of correlation, e. g. as derived from the proper motions, seems not to exceed 0.30 or 0.40, which value corresponds to $\lambda_1 = 0.91$ and 0.84 .² On the other hand, when the stars are separated in spectral classes, a larger coefficient of correlation results. Thus GYLLENBERG finds for stars of earlier spectral classes correlation-coefficients varying from 0.39 to 0.71, although here the exceptional fact may be considered that the stars in question are restricted to the apparently very bright stars. In the present paper a correlation-coefficient varying from 0.4 to 0.8 was found. The corresponding numerical values of λ_1 are 0.84 and 0.36.

In these three different expressions for the corrections to be applied to the absolute magnitude, two separate quantities are to be distinguished:

the *first* on account of the conversion of $\overline{1/r}$ to $\bar{r}$;

the *second* an effect caused by the dispersion in absolute magnitude, when transferring M_m into M_0 .

The simplest case is presented by the assumption of constant density in space. Introducing more complicated density functions, we arrive at similar, although slightly more complicated, expressions for the corrections.

A table has been calculated below (Table 19), giving the mean parallax, $\bar{\pi}$, the mean distance, $\bar{r}$, and the mean distance modulus, $\overline{5 \log r}$, when the parallactic motion in seconds of arc is known.

The quantities given in Table 19 are the *uncorrected* values. They have been computed for four different values of the solar velocity. When the parallactic

² Lund Medd Ser II No. 41 a (1925).

TABLE 19.
Table of the distance-modulus.

	$S = 15$ km/sec			$S = 20$ km/sec			$S = 25$ km/sec			$S = 30$ km/sec		
$\vartheta_1 S$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m - M$
"0005	"00016	6200.	14.01	"00012	8300.	14.63	"00009	11100.	15.12	"00008	12500.	15.51
.0006	.00019	5300.	13.61	.00014	7100.	14.24	.00011	9100.	14.72	.00009	11100.	15.12
.0007	.00022	4500.	13.28	.00017	5900.	13.90	.00013	7700.	14.38	.00011	9100.	14.78
.0008	.00025	4000.	12.99	.00019	5300.	13.61	.00015	6700.	14.10	.00013	7700.	14.49
.0009	.00028	3600.	12.73	.00021	4800.	13.36	.00017	5900.	13.84	.00014	7100.	14.24
.0010	.00032	3120.	12.50	.00024	4170.	13.13	.00019	5260.	13.61	.00016	6250.	14.01
.0011	.00035	2860.	12.30	.00026	3850.	12.92	.00021	4760.	13.40	.00017	5880.	13.80
.0012	.00038	2630.	12.11	.00028	3570.	12.73	.00023	4350.	13.21	.00019	5260.	13.61
.0013	.00041	2440.	11.93	.00031	3230.	12.56	.00025	4000.	13.04	.00021	4760.	13.44
.0014	.00044	2270.	11.77	.00033	3030.	12.40	.00027	3700.	12.88	.00022	4550.	13.28
.0015	.00047	2130.	11.62	.00036	2780.	12.25	.00028	3570.	12.73	.00024	4170.	13.13
.0016	.00051	1960.	11.48	.00038	2630.	12.11	.00030	3330.	12.59	.00025	4000.	12.99
.0017	.00054	1850.	11.35	.00040	2500.	11.97	.00032	3120.	12.46	.00027	3700.	12.85
.0018	.00057	1750.	11.23	.00043	2330.	11.85	.00034	2940.	12.33	.00028	3570.	12.73
.0019	.00060	1670.	11.11	.00045	2220.	11.73	.00036	2780.	12.22	.00030	3330.	12.61
.0020	.00063	1590.	11.00	.00047	2130.	11.62	.00038	2630.	12.11	.00032	3120.	12.50
.0021	.00066	1520.	10.89	.00050	2000.	11.52	.00040	2500.	12.00	.00033	3030.	12.40
.0022	.00069	1450.	10.79	.00052	1920.	11.41	.00042	2380.	11.90	.00035	2860.	12.29
.0023	.00073	1370.	10.69	.00054	1850.	11.32	.00044	2270.	11.80	.00036	2780.	12.20
.0024	.00076	1320.	10.60	.00057	1750.	11.23	.00045	2220.	11.71	.00038	2630.	12.11
.0025	.00079	1270.	10.51	.00059	1690.	11.14	.00047	2130.	11.62	.00039	2560.	12.02
.0026	.00082	1220.	10.43	.00062	1610.	11.05	.00049	2040.	11.54	.00041	2440.	11.93
.0027	.00085	1180.	10.34	.00064	1560.	10.97	.00051	1960.	11.45	.00043	2330.	11.85
.0028	.00088	1140.	10.27	.00066	1520.	10.89	.00053	1890.	11.37	.00044	2270.	11.77
.0029	.00092	1090.	10.19	.00069	1450.	10.81	.00055	1820.	11.30	.00046	2170.	11.69
.0030	.00095	1050.	10.12	.00071	1410.	10.74	.00057	1750.	11.23	.00047	2130.	11.62
.0031	.00098	1020.	10.05	.00073	1370.	10.67	.00059	1690.	11.15	.00049	2040.	11.55
.0032	.00101	990.	9.98	.00076	1320.	10.60	.00061	1640.	11.09	.00051	1960.	11.48
.0033	.00104	961.	9.91	.00078	1280.	10.53	.00063	1590.	11.02	.00052	1920.	11.41
.0034	.00107	935.	9.84	.00081	1230.	10.47	.00064	1560.	10.95	.00054	1850.	11.35
.0035	.00111	901.	9.78	.00083	1200.	10.41	.00066	1520.	10.89	.00055	1820.	11.29
.0036	.00114	877.	9.72	.00085	1180.	10.35	.00068	1470.	10.83	.00057	1750.	11.23
.0037	.00117	855.	9.66	.00088	1140.	10.29	.00070	1430.	10.77	.00058	1720.	11.17
.0038	.00120	833.	9.60	.00090	1110.	10.23	.00072	1390.	10.71	.00060	1670.	11.11
.0039	.00123	813.	9.55	.00092	1090.	10.17	.00074	1350.	10.66	.00062	1610.	11.05
.0040	.00126	794.	9.49	.00095	1050.	10.12	.00076	1320.	10.60	.00063	1590.	11.00
.0041	.00130	769.	9.44	.00097	1030.	10.06	.00078	1280.	10.55	.00065	1540.	10.94
.0042	.00133	752.	9.39	.00099	1010.	10.01	.00080	1250.	10.49	.00066	1520.	10.89
.0043	.00136	735.	9.33	.00102	980.	9.96	.00081	1230.	10.44	.00068	1470.	10.84
.0044	.00139	719.	9.28	.00104	961.	9.91	.00083	1200.	10.39	.00069	1450.	10.79
.0045	.00142	704.	9.24	.00107	935.	9.86	.00085	1180.	10.34	.00071	1410.	10.74
.0046	.00145	690.	9.19	.00109	917.	9.81	.00087	1150.	10.30	.00073	1370.	10.69
.0047	.00148	676.	9.14	.00111	901.	9.77	.00089	1120.	10.25	.00074	1350.	10.65
.0048	.00152	658.	9.10	.00114	877.	9.72	.00091	1100.	10.20	.00076	1320.	10.60
.0049	.00155	645.	9.05	.00116	862.	9.68	.00093	1080.	10.16	.00077	1300.	10.56

	S = 15 km/sec			S = 20 km/sec			S = 25 km/sec			S = 30 km/sec		
$\vartheta_1 S$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m-M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m-M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m-M$	$\bar{\pi} = \vartheta_1$	$\bar{r}$ parsecs	$m-M$
			<i>M</i>			<i>M</i>			<i>M</i>			<i>M</i>
.0050	.00158	633.	9.01	.00118	847.	9.63	.00095	1050.	10.12	.00079	1270.	10.51
.0051	.00161	621.	8.96	.00121	826.	9.59	.00097	1030.	10.07	.00080	1250.	10.47
.0052	.00164	610.	8.92	.00123	813.	9.55	.00099	1010.	10.03	.00082	1220.	10.43
.0053	.00167	599.	8.88	.00126	794.	9.51	.00100	1000.	9.99	.00084	1190.	10.39
.0054	.00171	585.	8.84	.00128	781.	9.47	.00102	980.	9.95	.00085	1180.	10.35
.0055	.00174	575.	8.80	.00130	769.	9.43	.00104	961.	9.91	.00087	1150.	10.31
.0056	.00177	565.	8.76	.00133	752.	9.39	.00106	943.	9.87	.00088	1140.	10.27
.0057	.00180	556.	8.72	.00135	741.	9.35	.00108	926.	9.83	.00090	1110.	10.23
.0058	.00183	546.	8.68	.00137	730.	9.31	.00110	909.	9.79	.00092	1090.	10.19
.0059	.00186	538.	8.65	.00140	714.	9.27	.00112	893.	9.76	.00093	1080.	10.15
.0060	.00190	526.	8.61	.00142	704.	9.24	.00114	877.	9.72	.00095	1050.	10.12
.0061	.00193	518.	8.58	.00145	690.	9.20	.00116	862.	9.68	.00096	1040.	10.08
.0062	.00196	510.	8.54	.00147	680.	9.16	.00117	855.	9.65	.00098	1020.	10.04
.0063	.00199	502.	8.50	.00149	671.	9.13	.00119	840.	9.61	.00099	1010.	10.01
.0064	.00202	495.	8.47	.00152	658.	9.10	.00121	826.	9.58	.00101	990.	9.98
.0065	.00205	488.	8.44	.00154	649.	9.06	.00123	813.	9.55	.00103	971.	9.94
.0066	.00209	478.	8.40	.00156	641.	9.03	.00125	800.	9.51	.00104	961.	9.91
.0067	.00212	472.	8.37	.00159	629.	9.00	.00127	787.	9.48	.00106	943.	9.88
.0068	.00215	465.	8.34	.00161	621.	8.96	.00129	775.	9.45	.00107	935.	9.84
.0069	.00218	459.	8.31	.00163	613.	8.93	.00131	763.	9.42	.00109	917.	9.81
.0070	.00221	452.	8.28	.00166	602.	8.90	.00133	752.	9.38	.00111	901.	9.78
.0071	.00224	446.	8.25	.00168	595.	8.87	.00135	741.	9.35	.00112	893.	9.75
.0072	.00227	440.	8.22	.00171	585.	8.84	.00136	735.	9.32	.00114	877.	9.72
.0073	.00231	433.	8.19	.00173	578.	8.81	.00138	725.	9.29	.00115	870.	9.69
.0074	.00234	427.	8.16	.00175	571.	8.78	.00140	714.	9.27	.00117	855.	9.66
.0075	.00237	422.	8.13	.00178	562.	8.75	.00142	704.	9.24	.00118	847.	9.63
.0076	.00240	417.	8.10	.00180	556.	8.72	.00144	694.	9.21	.00120	833.	9.60
.0077	.00243	411.	8.07	.00182	549.	8.69	.00146	685.	9.18	.00122	820.	9.57
.0078	.00246	406.	8.04	.00185	540.	8.67	.00148	676.	9.15	.00123	813.	9.55
.0079	.00250	400.	8.01	.00187	535.	8.64	.00150	667.	9.12	.00125	800.	9.52
.0080	.00253	395.	7.99	.00189	529.	8.61	.00152	658.	9.10	.00126	794.	9.49
.0081	.00256	391.	7.96	.00192	521.	8.58	.00153	654.	9.07	.00128	781.	9.46
.0082	.00259	386.	7.93	.00194	515.	8.56	.00155	645.	9.04	.00129	775.	9.44
.0083	.00262	382.	7.91	.00197	508.	8.53	.00157	637.	9.02	.00131	763.	9.41
.0084	.00265	377.	7.88	.00199	502.	8.51	.00159	629.	8.99	.00133	752.	9.39
.0085	.00268	373.	7.85	.00201	497.	8.48	.00161	621.	8.96	.00134	746.	9.36
.0086	.00272	368.	7.83	.00204	490.	8.45	.00163	613.	8.94	.00136	735.	9.33
.0087	.00275	364.	7.80	.00206	485.	8.43	.00165	606.	8.91	.00137	730.	9.31
.0088	.00278	360.	7.78	.00208	481.	8.40	.00167	599.	8.89	.00139	719.	9.28
.0089	.00281	356.	7.75	.00211	474.	8.38	.00169	592.	8.86	.00141	709.	9.26
.0090	.00284	352.	7.73	.00213	469.	8.36	.00171	585.	8.84	.00142	704.	9.24
.0091	.00288	347.	7.71	.00216	463.	8.33	.00172	581.	8.82	.00144	694.	9.21
.0092	.00291	344.	7.68	.00218	459.	8.31	.00174	575.	8.79	.00145	690.	9.19
.0093	.00294	340.	7.66	.00220	454.	8.28	.00176	568.	8.77	.00147	680.	9.16
.0094	.00297	337.	7.64	.00223	448.	8.26	.00178	562.	8.75	.00148	676.	9.14
.0095	.00300	333.	7.61	.00225	444.	8.24	.00180	556.	8.72	.00150	667.	9.12
.0096	.00303	330.	7.59	.00227	440.	8.22	.00182	549.	8.70	.00152	658.	9.10
.0097	.00307	326.	7.57	.00230	435.	8.19	.00184	543.	8.68	.00153	654.	9.07
.0098	.00310	323.	7.55	.00232	431.	8.17	.00186	538.	8.65	.00155	645.	9.05
.0099	.00313	319.	7.52	.00235	425.	8.15	.00188	532.	8.63	.00156	641.	9.03

	$S = 15$ km/sec			$S = 20$ km/sec			$S = 25$ km/sec			$S = 30$ km/sec		
$\vartheta_1 S$	$\bar{\pi} - \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} - \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} - \vartheta_1$	$\bar{r}$ parsecs	$m - M$	$\bar{\pi} - \vartheta_1$	$\bar{r}$ parsecs	$m - M$
"0100	"00316	316.5	7.50	"00237	421.9	8.13	"00189	529.1	8.61	"00158	632.9	9.01
.0110	.00347	288.2	7.30	.00261	383.1	7.92	.00208	480.8	8.40	.00174	574.7	8.80
.0120	.00379	263.9	7.11	.00284	352.1	7.73	.00227	440.5	8.21	.00189	529.1	8.61
.0130	.00411	243.3	6.93	.00308	324.7	7.56	.00246	406.5	8.04	.00205	487.8	8.44
.0140	.00442	226.2	6.77	.00332	301.2	7.40	.00265	377.4	7.88	.00221	452.5	8.28
.0150	.00474	211.0	6.62	.00355	281.7	7.25	.00284	352.1	7.73	.00237	421.9	8.13
.0160	.00505	198.0	6.48	.00379	263.9	7.11	.00303	330.0	7.59	.00253	395.3	7.99
.0170	.00537	186.2	6.35	.00403	248.1	6.97	.00322	310.6	7.46	.00268	373.1	7.85
.0180	.00569	175.7	6.23	.00426	234.7	6.85	.00341	293.3	7.33	.00284	352.1	7.73
.0190	.00600	166.7	6.11	.00450	222.2	6.73	.00360	277.8	7.22	.00300	333.3	7.61
.0200	.00632	158.2	6.00	.00474	211.0	6.62	.00379	263.9	7.11	.00316	316.5	7.50
.0210	.00663	150.8	5.89	.00497	201.2	6.52	.00398	251.3	7.00	.00332	301.2	7.40
.0220	.00695	143.9	5.79	.00521	191.9	6.41	.00417	239.8	6.90	.00347	288.2	7.29
.0230	.00727	137.6	5.69	.00545	183.5	6.32	.00436	229.4	6.80	.00363	275.5	7.20
.0240	.00758	131.9	5.60	.00569	175.7	6.23	.00455	219.8	6.71	.00379	263.9	7.11
.0250	.00790	126.6	5.51	.00592	168.9	6.14	.00474	211.0	6.62	.00395	253.2	7.02
.0260	.00821	121.8	5.43	.00616	162.3	6.05	.00493	202.8	6.54	.00410	243.9	6.93
.0270	.00853	117.2	5.34	.00640	156.2	5.97	.00512	195.3	6.45	.00426	234.7	6.85
.0280	.00885	113.0	5.27	.00663	150.8	5.89	.00531	188.3	6.38	.00442	226.2	6.77
.0290	.00916	109.2	5.19	.00687	145.6	5.81	.00549	182.1	6.30	.00458	218.3	6.69
.0300	.00948	105.5	5.12	.00711	140.6	5.74	.00568	176.1	6.23	.00474	211.0	6.62
.0310	.00979	102.1	5.05	.00734	136.2	5.67	.00587	170.4	6.15	.00489	204.5	6.55
.0320	.01011	98.91	4.98	.00758	131.9	5.60	.00606	165.0	6.09	.00505	198.0	6.48
.0330	.01042	95.97	4.91	.00782	127.9	5.53	.00625	160.0	6.02	.00521	191.9	6.41
.0340	.01074	93.11	4.84	.00805	124.2	5.47	.00644	155.3	5.95	.00537	186.2	6.35
.0350	.01106	90.42	4.78	.00829	120.6	5.41	.00663	150.8	5.89	.00553	180.8	6.29
.0360	.01137	87.95	4.72	.00853	117.2	5.35	.00682	146.6	5.83	.00568	176.1	6.23
.0370	.01169	85.54	4.66	.00876	114.2	5.29	.00701	142.7	5.77	.00584	171.2	6.17
.0380	.01200	83.33	4.60	.00900	111.1	5.23	.00720	138.9	5.71	.00600	166.7	6.11
.0390	.01232	81.17	4.55	.00924	108.2	5.17	.00739	135.3	5.66	.00616	162.3	6.05
.0400	.01264	79.11	4.49	.00948	105.5	5.12	.00758	131.9	5.60	.00632	158.2	6.00
.0425	.01343	74.46	4.36	.01007	99.30	4.99	.00805	124.2	5.47	.00671	149.0	5.87
.0450	.01421	70.37	4.24	.01066	93.81	4.86	.00853	117.2	5.34	.00710	140.8	5.74
.0475	.01500	66.67	4.12	.01125	88.89	4.74	.00900	111.1	5.23	.00750	133.3	5.62
.0500	.01579	63.33	4.01	.01184	84.46	4.63	.00947	105.6	5.12	.00789	126.7	5.51
.0525	.01659	60.28	3.90	.01244	80.39	4.53	.00995	100.5	5.01	.00829	120.6	5.41
.0550	.01737	57.57	3.80	.01303	76.75	4.43	.01042	95.97	4.91	.00868	115.2	5.31
.0575	.01816	55.07	3.70	.01362	73.42	4.33	.01089	91.83	4.81	.00908	110.1	5.21
.0600	.01895	52.77	3.61	.01421	70.37	4.24	.01137	87.95	4.72	.00947	105.6	5.12
.0650	.02053	48.71	3.44	.01540	64.94	4.06	.01232	81.17	4.55	.01026	97.47	4.94
.0700	.02211	45.23	3.28	.01658	60.31	3.90	.01326	75.41	4.39	.01105	90.50	4.78
.0750	.02369	42.21	3.13	.01777	56.27	3.75	.01421	70.37	4.24	.01184	84.46	4.63
.0800	.02527	39.57	2.99	.01895	52.77	3.61	.01516	65.96	4.10	.01263	79.18	4.49
.0850	.02685	37.24	2.85	.02014	49.65	3.48	.01611	62.07	3.96	.01342	74.52	4.36
.0900	.02843	35.17	2.73	.02132	46.90	3.36	.01705	58.65	3.84	.01421	70.37	4.24
.0950	.03001	33.32	2.61	.02251	44.42	3.24	.01800	55.56	3.72	.01500	66.67	4.12
.1000	.03159	31.66	2.50	.02369	42.21	3.13	.01895	52.77	3.61	.01579	63.33	4.01

motion for a group of stars (or for a single star) is calculated, the three quantities mentioned can be obtained directly from the table.

The values in Table 19 ought to be corrected for σ_M by means of Table 20 a. Here the corrections required by different assumptions of the space-density function have been computed for $\sigma_M = 0.5$ to $\sigma_M = 2.5$.

TABLE 20 a.

σ_M	Corr. to M_m for σ_M		Corr. to M_m to find M_0	
	$\frac{1}{2} b \sigma_M^2$	$\frac{5}{2} b \sigma_M^2$	$3 b \sigma_M^2$	$2 b \sigma_M^2$
0.50	0.058	0.288	0.345	0.230
.60	.083	.414	.497	.332
.70	.113	.564	.677	.451
.80	.147	.737	.884	.589
.90	.186	.932	1.119	.746
1.00	.230	1.151	1.381	.921
.10	.279	1.393	1.672	1.114
.20	.331	1.658	1.989	1.326
.30	.389	1.946	2.335	1.556
.40	.451	2.256	2.708	1.805
1.50	.518	2.590	3.108	2.072
.60	.589	2.947	3.537	2.358
.70	.665	3.327	3.993	2.662
.80	.746	3.730	4.476	2.984
.90	.831	4.156	4.987	3.325
2.00	.921	4.605	5.526	3.684
.10	1.015	5.077	6.092	4.062
.20	1.114	5.572	6.686	4.458
.30	1.218	6.090	7.308	4.872
.40	1.326	6.631	7.957	5.305
2.50	1.439	7.195	8.634	5.756

Table 20 b.

Table of the factor $e^{b^2 \sigma^2 M}$.

σ_M	$e^{b^2 \sigma^2 M}$
0	1
0.5	1.05
1.0	1.23
1.5	1.61
2.0	2.33
2.5	3.76

For example, if we have $\vartheta_1 S = .0120$ and assume the solar motion to be 20 km/sec, we find from Table 19 a distance modulus of 7.7. This gives, if the stars have an apparent magnitude 8.5, an absolute magnitude of $M_m = +0.8$. Applying

the correction for the dispersion $\sigma_M = 1^m0$, we find $M_m = +0^m6$. For the mean parallax we obtain $\bar{\pi} = .00284$, and the mean distance is $r_m = 352$ parsecs. The latter quantity ought to be corrected for the dispersion σ_M by the factor 1.23. Hence we find: $r_m = 433$ parsecs.

In the case of the irregular variables attention should be paid to the importance of the error in amplitude, or in the maxima and minima used when computing the photometrical factor $10^{0.2m}$. The uncertainty in the limiting magnitudes affects the reduced proper motions through the photometrical factor.

In the present investigation the data given in PRAGER: »Katalog und Ephemeriden Veränderlicher Sterne«, have been used. In dealing with variables with regular light-curves and fairly constant amplitudes, like Cepheids and long-period variables, these considerations may be superfluous. The light-curves of the irregular variables, however, are very complicated. The amplitude varies, and the recorded values of the maximum and minimum magnitudes might possibly not represent the real phases. Because of the complexity of the light-curves of these variables, the amplitude as obtained from the curve is not constant. On the contrary it varies considerably, hence giving varying limits to the brightness, when measured in different parts of the curve. Only an analysis of the several periods in the variation of light would give the true limiting magnitudes. The values observed are the result of the interfering periods.

An expression for the error in reduced proper motion caused by an error in m can easily be derived. It is 1) due to interference in the part of the curve in question, 2) a dispersion in the star-light itself, 3) an observational error. Of course, in reality all three sources contribute to the error. At present we will presume the error to be a general one. The part of the error due to the phase is in fact a periodical error, linked with the original interfering periods of light-variation. Since we cannot at present deduce these periods except in very few cases, we are not able to correct the magnitude for its periodical variation. Therefore the present consideration will have, in reality, only theoretical interest.

By m we denote the true apparent magnitude at a given phase. The error in this magnitude $= \Delta m$.

Further:

$$\begin{aligned} u &= \text{unreduced proper motion} \\ u' &= \text{true reduced proper motion} \\ \omega' &= \begin{cases} \text{observed} \\ \text{apparent proper motion} \end{cases} \\ \Delta u &= \text{error in reduced proper motion.} \end{aligned}$$

Since:

$$u' = 10^{0.2m}u, \quad (116)$$

and:

$$u' + \Delta u' = \omega', \quad (117)$$

we obtain.

$$10^{0.2(m+\Delta m)} u = u' + \Delta u' = \omega'. \quad (118)$$

We write:

$$10^{0.2m} u \ 10^{0.2\Delta m} = \omega'.$$

Then:

$$u' \ 10^{0.2\Delta m} = \omega'. \quad (119)$$

The quantity ω' is known from the observations. We obtain the unknown true reduced proper motion u' from:

$$u' = \omega' \ 10^{-0.2\Delta m} = \omega' e^{-b\Delta m}. \quad (120)$$

If Δm is known, we are able to determine u' . It might be convenient to calculate the correcting factor in the form of a table from the argument Δm .

In the present case the difficulty is that Δm is mostly an unknown quantity.

$$\begin{array}{ll} \text{For: } \Delta m > 0, & \text{For: } \Delta m < 0, \\ u' < \omega'. & u' > \omega'. \end{array}$$

The error in the absolute magnitude obtained from the reduced proper motions is easily shown to be equal to $-\Delta m$.

Numerical values of the absolute magnitude.

By the different treatments of the proper motions the parallax motion $\vartheta_1 S$ or $\theta_1 S$ has been obtained. Introducing the value of the solar motion, as obtained from the radial velocities, the mean parallax and mean absolute magnitude have been computed.

To give a survey of the solutions performed with the present material, the following scheme has been constructed.

I. Material: all stars up to $m_{\max} = 11.5$.

A. Apex unknown. Non-reduced and reduced proper motions.

- 1) The material has been separated into *twelve* groups with reference to magnitude (half-magnitude intervals). The parallax motion has been computed for each of the twelve groups.
- 2) The material has been grouped into *seven* groups with the object of obtaining as nearly as possible the same number of stars in each group. The solutions are worked out as above.

B. Apex known.

Since the coordinates of apex obtained from the solution of the radial velocities agree well with Standard Apex, apex's position was regarded as known. The direction-cosines c_{ij} referring the stars to this point have been computed, and the mean parallax and absolute magnitude were derived from these conditions. The solution was worked out for the groups

TABLE

A p e x u n k n o w n													
$m_{\max}$	$\bar{m}$	N	N o n - r e d u c e d p r o p e r m o t i o n s										
			$\theta_1 S$	r (parsecs)					M_m				
				$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
2.0— 4.9	4.00	6	.0413	105	111	129	169	246	— 1.11	— 1.17	— 1.35	— 1.64	— 2.03
5.0— 6.4	5.64	10	.0236	184	194	227	297	431	— 0.69	— 0.75	— 0.92	— 1.21	— 1.61
6.5— 6.9	6.76	9	.0339	128	135	158	207	300	+ 1.22	+ 1.16	+ 0.99	+ 0.70	+ 0.30
7.0— 7.4	7.17	8	.0559	78	82	96	125	182	+ 2.71	+ 2.65	+ 2.48	+ 2.19	+ 1.79
7.5— 7.9	7.70	10	.0262	166	175	204	268	388	+ 1.60	+ 1.54	+ 1.37	+ 1.08	+ 0.68
8.0— 8.4	8.23	22	.0236	184	194	227	297	430	+ 1.90	+ 1.84	+ 1.67	+ 1.38	+ 0.98
8.5— 8.9	8.67	33	.0120	361	381	445	583	844	+ 0.88	+ 0.82	+ 0.65	+ 0.36	— 0.04
9.0— 9.4	9.20	28	.0096	452	476	556	728	1055	+ 0.93	+ 0.87	+ 0.70	+ 0.41	+ 0.01
9.5— 9.9	9.69	37	.0113	384	405	473	619	897	+ 1.77	+ 1.71	+ 1.54	+ 1.25	+ 0.85
10.0—10.4	10.21	31	.0145	301	317	370	484	702	+ 2.82	+ 2.76	+ 2.59	+ 2.30	+ 1.90
10.5—10.9	10.67	18	.0146	297	313	366	479	694	+ 3.30	+ 3.24	+ 3.07	+ 2.78	+ 2.38
11.0—11.4	11.19	7	.0177	246	259	302	396	574	+ 4.24	+ 4.18	+ 4.01	+ 3.72	+ 3.32

mentioned above under 1) and 2), and was performed with both non-reduced and reduced proper motions.

II. Proper motions for stars fainter than $m_{\max} = 9.9$ rejected. The seven groups defined in I A 2) were treated as follows: — Groups VI and VII were omitted. I and II were taken together as a new group, and III, IV and V as another. In order to do justice to the greater accuracy in the proper motions of the bright stars, in forming the groups different weights were applied to the sub-groups. Thus I and II were attributed the weight 1, III was also given the weight 1, but IV and V the weight $\frac{1}{2}$. It was intended to assign a certain weight, depending on the mean errors, to each individual proper motion. This was, however, not considered to be necessary, the weight being more or less insignificant. In the computations, therefore, all proper motions have been regarded as having equal weight.

III. Proper motions for stars of magnitudes $5.0 \leq m_{\max} \leq 9.4$.

Two groups were formed, separated according to the galactic latitude of $\pm 15^\circ$. The solutions were worked out in the system of both Boss and FK3.

The values of the absolute magnitudes of the irregular variables, derived by the methods sketched above, are given below in tables 21 to 24.

21 a.

A p e x u n k n o w n													
$m_{\max}$	$\bar{m}$	N	R e d u c e d p r o p e r m o t i o n s										
			$\vartheta_1 S$	R (parsecs)					M_m				
				$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
2.0 — 4.9	4.00	6	.2473	17.6	18.5	21.6	28.3	41.1	— 1.23	— 1.29	— 1.46	— 1.75	— 2.15
5.0 — 6.4	5.64	10	.3609	12.0	12.7	14.8	19.4	28.1	— 0.41	— 0.47	— 0.64	— 0.93	— 1.33
6.5 — 6.9	6.76	9	.7612	5.7	6.0	7.0	9.2	13.3	+ 1.22	+ 1.16	+ 0.99	+ 0.70	+ 0.30
7.0 — 7.4	7.17	8	1.520	2.9	3.0	3.5	4.6	6.7	+ 2.71	+ 2.65	+ 2.48	+ 2.19	+ 1.79
7.5 — 7.9	7.70	10	.8610	5.1	5.3	6.2	8.1	11.8	+ 1.48	+ 1.42	+ 1.25	+ 0.96	+ 0.56
8.0 — 8.4	8.23	22	1.093	4.0	4.2	4.9	6.4	9.3	+ 2.00	+ 1.94	+ 1.77	+ 1.48	+ 1.08
8.5 — 8.9	8.67	33	.6034	7.2	7.6	8.9	11.6	16.8	+ 0.71	+ 0.65	+ 0.48	+ 0.19	— 0.21
9.0 — 9.4	9.20	28	.7040	6.2	6.5	7.6	9.9	14.4	+ 1.04	+ 0.98	+ 0.81	+ 0.52	+ 0.12
9.5 — 9.9	9.69	37	.9719	4.5	4.7	5.5	7.2	10.4	+ 1.74	+ 1.68	+ 1.51	+ 1.22	+ 0.82
10.0 — 10.4	10.21	31	1.382	3.1	3.3	3.9	5.1	7.3	+ 2.51	+ 2.45	+ 2.28	+ 1.99	+ 1.59
10.5 — 10.9	10.67	18	1.847	2.4	2.5	2.9	3.8	5.5	+ 3.14	+ 3.08	+ 2.91	+ 2.62	+ 2.22
11.0 — 11.4	11.19	7	3.499	1.2	1.3	1.5	2.0	2.9	+ 4.52	+ 4.46	+ 4.29	+ 4.00	+ 3.60

The corrections on account of the dispersion in absolute magnitude have been applied with regard to four different values of this quantity: $\sigma_M=0^m5$, 1^m0 , 1^m5 and 2^m0 . The attempts made to determine the numerical value of σ_M for the irregular variables indicate it to be close to 1^m5 . The values of M and r , which have been derived by use of the corrections for this value of the dispersion, have always been used in the following discussions. The corrections required increase considerably with increasing σ_M . When no account is taken of this correction, the absolute magnitudes prove too faint.

The mean errors have been derived for the average material. Thus we have:

$$\begin{aligned}\varepsilon(\vartheta_1 S) &= \pm .0028, \\ \varepsilon(M) &= \pm 0^m40, \\ \varepsilon(r) &= \pm 11 \text{ parsecs.}\end{aligned}$$

We obtain a mean value of the absolute magnitude for the total present material of:

$$M_m = + 0^m35 \pm 0^m40.$$

It is of interest to consider the values of M_m in Table 23. The material of proper motions has in this solution been divided at $b = \pm 15^\circ$. The mean parallax and the mean absolute magnitude have been determined for galactic stars and

TABLE

A p p e x k n o w n													
$m_{\max}$	$\bar{m}$	N	N o n - r e d u c e d p r o p e r m o t i o n s										
			$\vartheta_1 S_{\frac{r}{r_0}}$	r (parsecs)					M_m				
				$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
2.0—4.9	4.00	6	.0243	179	188	221	288	417	—2.26	—2.32	—2.49	—2.78	—3.18
5.0—6.4	5.64	10	.0019	2348	2476	2889	3784	5485	—6.22	—6.26	—6.45	—6.74	—7.14
6.5—6.9	6.76	9	.0314	139	146	171	223	324	+1.05	+0.99	+0.82	+0.53	+0.13
7.0—7.4	7.17	8	.0455	96	101	118	154	223	+2.27	+2.21	+2.04	+1.75	+1.35
7.5—7.9	7.70	10	.0252	172	182	212	278	403	+1.52	+1.48	+1.29	+1.00	+0.60
8.0—8.4	8.23	22	.0235	185	195	228	298	432	+1.89	+1.83	+1.66	+1.37	+0.97
8.5—8.9	8.67	33	.0120	363	383	447	586	849	+0.87	+0.81	+0.64	+0.35	—0.05
9.0—9.4	9.20	28	.0032	1380	1455	1698	2224	3223	—1.50	—1.56	—1.73	—2.02	—2.42
9.5—9.9	9.69	37	.0007	6022	6350	7410	9703	14064	—4.21	—4.27	—4.44	—4.73	—5.13
10.0—10.4	10.21	31	.0073	597	629	734	961	1393	+1.33	+1.27	+1.10	+0.81	+0.41
10.5—10.9	10.67	18	.0093	467	492	574	752	1090	+2.32	+2.26	+2.09	+1.80	+1.40
11.0—11.4	11.19	7	.0182	239	252	294	384	557	+4.30	+4.24	+4.07	+3.78	+3.38

non-galactic stars separately. The galactic stars are found to be considerably brighter than the non-galactic ones. The variables lying in the galactic zone appear to be super-giants among the irregular variables. We shall in the following have occasion to study another group of possible super-giants. There seems to be a certain probability for the existence of groups of irregular variables with marked differences in absolute magnitude. This was also referred to in Chapter 6.

Absolute magnitudes from the galactic z -coordinates.

Starting from the definition of the galactic z -coordinates:

$$z = R 10^{0.2m} \sin b, \quad (121)$$

we may form the mean value z'_0 of $z' = \frac{z}{R}$, the average value $|\bar{z}'|$ of the same quantity, and the dispersion $\sigma_{z'}$.

From these three values we easily deduce the following expressions of the absolute magnitude of the group of stars:

$$M = 5 \log z'_0 + \text{const.} \quad (122 \text{ a})$$

$$M = 5 \log |\bar{z}'| + \text{const.} \quad (122 \text{ b})$$

$$M = 5 \log \sigma_{z'} + \text{const.} \quad (122 \text{ c})$$

21 b.

A p e x k n o w n													
$m_{\max}$	$\bar{m}$	N	R e d u c e d p r o p e r m o t i o n s										
			$\theta_1 s$	R (parsecs)					M_m				
				$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
2.0 — 4.9	4.00	6	.1627	26.7	28.2	32.9	43.1	62.4	— 2.14	— 2.20	— 2.37	— 2.66	— 3.06
5.0 — 6.4	5.64	10	.0772	56.4	59.4	69.4	90.8	131.6	— 3.76	— 3.82	— 3.99	— 4.28	— 4.68
6.5 — 6.9	6.76	9	.7013	6.2	6.5	7.6	10.0	14.5	+ 1.04	+ 0.98	+ 0.81	+ 0.52	+ 0.12
7.0 — 7.4	7.17	8	1.2172	3.6	3.8	4.4	5.8	8.3	+ 2.23	+ 2.17	+ 2.00	+ 1.71	+ 1.31
7.5 — 7.9	7.70	10	.8730	5.0	5.3	6.1	8.0	11.6	+ 1.51	+ 1.45	+ 1.28	+ 0.99	+ 0.59
8.0 — 8.4	8.23	22	1.0491	4.1	4.4	5.1	6.7	9.7	+ 1.91	+ 1.85	+ 1.68	+ 1.39	+ 0.99
8.5 — 8.9	8.67	33	.6015	7.2	7.6	8.9	11.6	16.9	+ 0.70	+ 0.64	+ 0.47	+ 0.18	— 0.22
9.0 — 9.4	9.20	28	.2821	15.4	16.2	18.9	24.8	35.9	— 0.94	— 1.00	— 1.17	— 1.46	— 1.86
9.5 — 9.9	9.69	37	.0588	74.0	78.0	91.1	119.2	172.8	— 4.35	— 4.41	— 4.58	— 4.87	— 5.27
10.0 — 10.4	10.21	31	.9609	4.5	4.8	5.6	7.3	10.6	+ 1.72	+ 1.66	+ 1.49	+ 1.20	+ 0.80
10.5 — 10.9	10.67	18	1.2145	3.6	3.8	4.4	5.8	8.4	+ 2.23	+ 2.17	+ 2.00	+ 1.71	+ 1.31
11.0 — 11.4	11.19	7	3.4227	1.3	1.3	1.6	2.0	3.0	+ 4.48	+ 4.42	+ 4.25	+ 3.96	+ 3.56

Evidently these methods of determining the absolute magnitude are relative methods. They have been used in this paper to study the absolute magnitude for stars of different spectral class.

Irregular variables in galactic star-clouds.¹

Another way to obtain a criterion of the absolute magnitudes would be to consider isolated groups of stars in special regions of the sky for which the distance is known, e. g. star-clouds. If these are found to contain Cepheids and irregular variables, the absolute magnitudes of the irregular variables should easily be derived.

From the galactic distribution it will be seen that the irregular variables, like the Cepheids and long-period variables, sometimes lie quite close together, forming groups or clusters. Such clusters are observed in Cygnus, Perseus, Orion, Scutum, Scorpius and other parts of the Milky Way. The Cepheids also cluster in these regions. Since the absolute magnitudes of the last mentioned variables can be obtained from the period-luminosity curve, the absolute magnitudes of the irregular variables can be deduced.

In view of the importance of extending our knowledge of the absolute magnitudes of the irregular variables, attempts were made to use this method, though the probability of finding the mutually connected stars is certainly small. Only stars

TABLE 22.

Table of the mean distances r and the mean absolute magnitudes M_m with the material parted into seven groups.

A p p e x u n k n o w n														
Gr.	$m_{\max}$	$\bar{m}$	N	N o n - r e d u c e d p r o p e r m o t i o n s										
				$\theta_1 S$	r (parsecs)					M_m				
					$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
I	2.0—7.4	5.89	33	.0281	155	163	191	250	362	—0.06	—0.12	—0.29	—0.58	—0.98
II	7.5—8.4	7.96	32	.0255	190	201	234	307	444	+1.26	+1.20	+1.03	+0.74	+0.34
III	8.5—8.9	8.67	33	.0120	361	381	445	583	844	+0.88	+0.82	+0.65	+0.36	—0.04
IV	9.0—9.4	9.20	28	.0096	452	476	556	728	1055	+0.93	+0.87	+0.70	+0.41	+0.01
V	9.5—9.9	9.69	37	.0113	384	405	473	619	897	+1.77	+1.71	+1.54	+1.25	+0.85
VI	10.0—10.4	10.21	31	.0145	301	317	370	484	702	+2.82	+2.76	+2.59	+2.30	+1.90
VII	10.5—11.5	10.93	27	.0153	284	299	349	457	663	+3.66	+3.60	+3.43	+3.14	+2.74

from the densest clusters have been considered. Of course, even in this case the stars may only be *apparently* near to each other. If, however, it were possible to find the correct distance-modulus, the method would open up possibilities for determining individual absolute magnitudes, which is very desirable.

For these attempts I have chosen the regions of Orion ($l=165^\circ$, $b=-5^\circ$), Cygnus ($l=41^\circ$, $b=+4^\circ$), Scutum ($l=355^\circ$, $b=-5^\circ$) and Perseus ($l=103^\circ$, $b=-2^\circ$). The Cepheids and irregular variables lying in these regions have been collected. The region has been graphically reproduced on a large scale, and variables of Cepheid and irregular class have been studied according to position. When two or more stars of the two classes were found to lie close together, they were tentatively assumed to be at the same distance from us. By means of the period of the Cepheid, which is known, the distance modulus was derived. In this way individual absolute magnitudes of the irregular variables were calculated.

The importance to be ascribed to these values must sometimes be questioned, since the whole proceeding as mentioned above is based on the assumption that the stars belong to the same cluster. Below is given a list of the stars whose absolute magnitudes have been obtained in this way.

Orion-region.

The following stars in the list of Cepheids have been considered.

From the Cepheids CR, CU, CV, DY Ori the mean distance-modulus $M - m = -14.99$ was derived.

The following irregular variables in the diagram have been considered usable

TABLE 23.

Apex unknown. Non-reduced proper motions.

$$[m_{\max} = 5.0-9.4].$$

a.

System: Boss.

b	$\overline{m}_{\max}$	$\vartheta_1 S$	r (parsecs)					M_m (max)					N
			$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	
$b \leq \pm 15^\circ$	8.23	.0046	940	991	1157	1515	2197	-1.64	-1.70	-1.87	-2.16	-2.56	51
$b > \pm 15^\circ$	7.97	.0241	180	190	222	290	421	+1.69	+1.63	+1.46	+1.17	+0.77	71
All	8.10	.0152	286	302	352	461	669	+0.81	+0.75	+0.58	+0.29	-0.11	122

b.

System: FK3.

b	$\overline{m}_{\max}$	$\vartheta_1 S$	r (parsecs)					M_m (max)					N
			$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	
$b \leq \pm 15^\circ$	8.23	.0087	500	527	615	806	1168	-0.27	-0.33	-0.50	-0.79	-1.19	51
$b > \pm 15^\circ$	7.97	.0202	215	227	264	346	502	+1.31	+1.25	+1.08	+0.79	+0.39	71
All	8.10	.0118	368	388	453	593	859	+0.27	+0.21	+0.04	-0.25	-0.65	122

for a determination of their absolute magnitudes by means of the said distance-modulus:

Irr	$m_{\max}$	$M_{\max}$
DQ Ori	12.3	-2.69
CX	13.0	-1.99
DP	10.3	-4.69
DV	13.2	-1.79
CT	10.9	-4.09
EE	12.8	-1.19

Here only variables very close in position to the Cepheids have been considered. The number would be increased if stars lying farther away from them were also taken into account.

Cygnus-region.

If the distance-modulus value of $-12^m.7$ obtained by W. BAADÉ³ is accepted for the stars of the star-cloud with its centre at $\alpha = 19^h 52^m.3$, $\delta = +38^\circ 13'$

³ ApJ 79.475 (1934).

TABLE
Table of the mean distance and the mean
[Values obtained from

Method of determination	$\bar{m}_{\max}$	N	$\vartheta_1 S$ $\Theta_1 S$
Apex unknown; non-reduced p.m.; all stars	8.26	224	.0128
» » ; » [strict solution]..	8.26	224	.0138
» » ; $m = 6.5-10.4$	8.45	181	.0147
» » ; $m = 6.5-11.4$	8.95	208	.0144
» » ; $m = 5.0-9.4$	8.10	122	.0152
» ; reduced p. m.; all stars	8.26	224	.8769
Apex known ; reduced p. m.; »	8.26	224	.6093

(1900.0), we obtain the following list of absolute magnitudes of irregular variables (column 3): —

Irr	$m_{\max}$	$M_{\max}$	$M_{\max}$
(1)	(2)	(3)	(4)
V ₃₈₅ Cyg	13.8	+1.1	
QZ	11.2	—1.5	
AX	7.4	—3.2	
AH	10.0	—1.0	
AA	8.4	—2.6	
NO	13.1	+0.4	
NR	13.5	+0.8	
OQ	13.6	+0.9	
OY	14.6	+1.9	
PR	14.7	+2.0	
PU	13.7	+1.0	
PZ	15.3	+2.6	
RR	12.9	+0.2	— 0.5
CO	9.4	—3.3	
V ₃₅₄	11.8	—0.9	
AZ	8.1	—2.9	
CY	9.9	—2.8	—3.45
AD	8.3	—2.7	—4.1
AI	7.8	—3.2	—4.6
V ₃₈₄	12.7	0.0	
V ₃₆₀	10.7	—0.3	

By means of individual Cepheids the value of $(M-m)$ may be found and used for irregular variables situated near to the Cepheid in question. In this way the values for the irregular variables of column 4 are obtained. Here the Cepheid BZ Cyg, the distance modulus of which is — 13.35, is considered. With this star are combined the irregular variables RR and CY Cyg. The Cepheid V₃₈₃ Cyg ($M-m = -14.1$) may be combined with the irregular variables AD and AI Cyg.

24.

absolute magnitude of the irregular variables.

the reduced proper motions].

r resp. R (parsecs)					$\overline{M}_m$				
$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
340	359	418	548	794	+0.60	+0.54	+0.37	+0.08	-0.32
317	334	390	510	740	+0.75	+0.69	+0.52	+0.23	-0.17
330	348	407	532	772	+0.85	+0.79	+0.62	+0.33	-0.07
303	319	372	488	707	+1.54	+1.48	+1.31	+1.02	+0.62
286	302	352	461	669	+0.81	+0.75	+0.58	+0.29	-0.11
5.0	5.2	6.1	8.0	11.6	+1.52	+1.46	+1.29	+1.00	+0.60
7.1	7.5	8.8	11.5	16.7	+0.73	+0.67	+0.50	+0.21	-0.19

Scutum-region.

From individual values of the Cepheids the following data have been obtained.

By means of the Cepheids RU and TY Sct — distance-modulus 13.90 — the absolute maximum magnitude of the irregular variable RR Sct is found equal to —5.70. In the same way the absolute magnitude of the irregular variable R Sct assumes the very high absolute magnitude —8.57 from the Cepheids BW and BX Sct ($M-m=-14.67$). From the Cepheid Y Sct ($M-m=-13.10$) we obtain the magnitude —3.40 for the irregular variable RX Sct.

The graph suggests that it is possible to take together all stars below $b=-10^\circ$. From the Cepheids the value of $M-m=-13.86$ (8 stars) is obtained. Hence we arrive at the following values of the absolute magnitudes of the irregular variables of this region:

Irr	$m_{\max}$	$M_{\max}$
RR Sct	8.3	-5.66
R	4.5	-7.76
RZ	7.5	-0.86
RX	9.7	-4.16
S	7.0	-4.76
T	8.8	-2.96
RT	9.1	-3.06

Perseus-region.

From the Cepheids UX, UY, VX and VY Per the mean distance-modulus 13.66 was calculated. By means of this value the absolute magnitudes of the irregular variables in the region were derived. The following values were found:

Irr	$m_{\max}$	$M_{\max}$
AD Per	7.7	— 4.26
SU	7.0	— 4.96
S	7.2	— 4.76
RS	8.0	— 3.96
T	8.0	— 4.06
YZ	9.8	— 4.36

In many cases the absolute magnitudes of individual irregular variables appear to be too bright. This occurs e. g. in the Scutum-region and in the Perseus-region. In the Scutum-region the variable R Sct has by the present method obtained an absolute magnitude of — 7.76, which must certainly be regarded as too bright. The value of the spectroscopical magnitude is about — 1.5. The remaining bright magnitudes might also be doubted. The longest series is apparently the one in Cygnus. The values here obtained agree on an average with those found by spectroscopic methods, and also with the mean value, obtained from the stars' parallactic motion.

More regions than those discussed above have been investigated, but the data appear to be questionable.

Irregular variables in globular clusters.

For stars in globular clusters the relation $M_1 - M_2 = m_1 - m_2$ would always be satisfied. However, very few irregular variables have been observed in globular clusters.

When investigating the periods and light-curves of variables in three clusters near the Small Magellanic Cloud, HELEN B. SAWYER⁴ found two irregular variables. One of them, catalogue no. 2 in the cluster NGC 362 on the edge of the Small Magellanic Cloud, has a period of 105^d22 with irregularities. The other star, no. 4, in the cluster NGC 6121 = M4, is reported as »probably irregular around a long period». The observed data of these two variables are given below:

	Var.	Instr.	Max	Min	Med	A	P
NGC 362	2		13 ^m 0	14 ^m 5	13 ^m 75	1 ^m 5	105 ^d 22 (irr)
NGC 6121	4	refr.	10.7	13.0	11.85	2.3	probably irr around a long period.
M	4	refl.	11.0	11.5			

Both variables are brighter than the cluster-variables. The first star is noted by Miss SAWYER to be 1^m7 brighter than the mean of the cluster variables. The second one is about 2^m brighter.

⁴ HC 366 (1931).

The variable 2 in NGC 362 has an absolute photographic magnitude of $-1^m.74$ and an absolute bolometric magnitude of $-4^m.80$. The spectrum is given as M0.

For the second variable no data are given. From data referring to another star (no. 29) we deduce the absolute photographic magnitude $-2^m.03$. These two stars' absolute magnitudes hence agree with the mean of the bright irregular variables in our Galaxy.

A case similar to that related above is represented by an irregular variable in M5 (no. 50), which star, as remarked by Miss SAWYER, has been found by BAILEY⁵ to have an average magnitude $1^m.2$ brighter than that of the cluster-type variables. This variable has a period of 106^d .

⁵ HA 78 (1917).

CHAPTER 11

Discussion of the absolute magnitudes

The absolute magnitudes of the irregular variables given above prove these stars to be absolutely very bright. In the period-luminosity diagram they are found to lie near the top-point of the curve, hence proving themselves to belong to the brightest of all variables in our Galaxy. Their mean magnitude corresponds to a mean period of about 100^d, or to a long period of about 600^d. The present material does not allow a period-luminosity relation to be determined, since the data are still too scanty and uncertain (Table 25).

TABLE 25.

Period—absolute magnitude relation.

Apex unknown. Non-reduced proper motions.

System: Boss.

P	$\bar{P}$	$\overline{\log P}$	$\overline{M}$ [$\sigma_M = 1.5$]	$\bar{m}$	N
$< 70^d$	53.9	1.732	— 0.28	7.75	12
70—100	84.5	1.927	+ 1.63	8.94	33
100—140	127.5	2.106	+ 1.00	9.75	20
140—200	169.7	2.230	+ 0.96	9.06	16
200—300	233.8	2.369	— 1.54	7.99	11
> 300	449.2	2.652	+ 1.36	8.92	10

Correlation between absolute and apparent magnitude.

The correlation between absolute and apparent magnitude has been studied from the statistically obtained values of Table 22. Fig. 31 gives the values of M_m corresponding to the mean values of m within the groups.

It is clear that there is a positive correlation.

At present the curve representing the correlation will be regarded as a straight line, viz. the regression-line, represented by the following equation:

$$M - M_0 = r_{M,m} \frac{\sigma_M}{\sigma_m} (m - m_0), \quad (123)$$

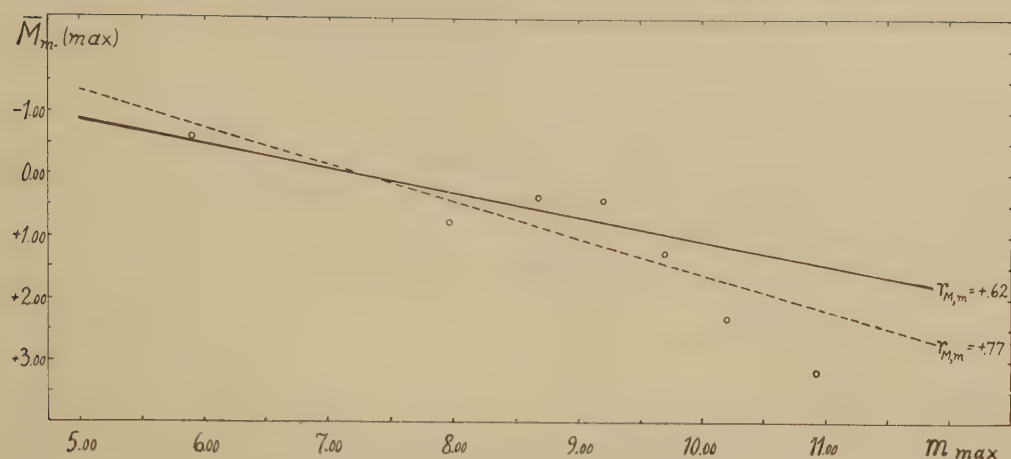


Fig. 31. Correlation between the absolute magnitude, obtained from the proper motions, and the apparent magnitude.

where: $r_{M,m}$ is the coefficient of correlation
 σ_M is the dispersion in absolute magnitude
 σ_m is the dispersion in apparent magnitude.

As has been pointed out in Chapter 6, we have

$$r_{M,m} = \frac{\sigma_M}{\sigma_m}. \quad (124)$$

Then our regression-line can be written:

$$M - M_0 = r_{M,m}^2 (m - m_0). \quad (125)$$

In order to determine the coefficient $r_{M,m}$ we have proceeded as follows.

1. The mean values of the seven groups of Table 22 were treated by the methods of least squares, and the regression-line was assumed to have the following form:

$$M = a m + b. \quad (126)$$

Consequently:

$$a = r_{M,m}^2. \quad (127)$$

In order to account for probably existing uncertainties in the proper motions and in the faint magnitudes, different weights were attributed to the points.

a) *All points were assigned equal weight.* With this solution the regression-line was obtained:

$$M = + 0.677 m - 4.958. \quad (128)$$

From here the following value of $r_{M,m}$ was derived:

$$r_{M,m} = +0.822 \pm 0.025. \quad (129)$$

b) *The weight = $1/2$ was attached to groups VI and VII, which represent the part of the material where the proper motions and the magnitudes are most uncertain. The weights of the other groups were as before taken equal to 1.*

The regression-line now has the form:

$$M = +0.589 m - 4.288, \quad (130)$$

and hence $r_{M,m}$ has the following value:

$$r_{M,m} = +0.767 \pm 0.030. \quad (131)$$

c) *Groups VI and VII are excluded. Groups I—V were given the same weight. In this case the following equation of the line of regression was found:*

$$M = +0.389 m - 2.786. \quad (132)$$

The coefficient of correlation is here:

$$r_{M,m} = +0.624 \pm 0.043. \quad (133)$$

As could be expected beforehand, the coefficient of correlation decreases when the weight is diminished for the groups containing stars of fainter magnitudes.

The mean value of the three coefficients of correlation, found above, is:

$$r_{M,m} = 0.738 \pm 0.033, \quad (134)$$

when the same weight is given to the three values.

However, as has been shown elsewhere in this paper, the clear decrease in the value of the coefficient $r_{M,m}$ may not with certainty be a step towards a more correct value of this quantity. The possible existence of two or more groups has been pointed out. Consequently a rather large value of the coefficient of correlation might be expected. The values found are higher than those obtained by the other methods and for other classes of stars. However, GYLLENBERG's ¹ investigation of the material of the Boss PGC gives a mean value of:

$$r_{M,m} = +0.60 \pm 0.04. \quad (135)$$

Later investigations indicate a lower value of $r_{M,m}$. Assuming the limits of σ_m to be $1^m.5$ to $3^m.0$, GYLLENBERG ² finds the following limits of $r_{M,m}$ for stars of earlier spectral classes:

$$+0.23 < r_{M,m} < +0.45. \quad (136)$$

¹ Lund Medd Ser I No. 142 (1936).

² Lund Medd Ser I No. 144 (1936).

2. From the list of stars with individual absolute magnitudes, the coefficient of correlation has been computed directly.

There was obtained:

$$r_{M,m} = +0.43 \pm 0.15. \quad (137)$$

Within the limits of error this value agrees with the one obtained in the above solution c. This value is an average value. There is an indication of one group of bright magnitudes having a large coefficient of correlation, and one of faint magnitudes having a smaller coefficient.

From the value of $r_{M,m}$ it is possible to obtain the dispersion in absolute magnitude, when the dispersion in apparent magnitude is known. This can be done by means of the known relation (124).

For the present material σ_m has been adopted equal to $2^m20 \pm 0^m06$. If we accept a larger σ_m , such as has been found for the stars in general, a larger value of σ_M results.

We obtain the following series of values:

$r_{M,m}$	σ_M		
	$\sigma_m = 2.20$	$\sigma_m = 3.00$	$\sigma_m = 4.00$
0.82	1 ^m 80	2 ^m 46	3 ^m 28
0.77	1.69	2.37	3.08
0.62	1.36	1.86	2.48
0.43	0.95	1.29	1.72

If we accept the larger values of the coefficient of correlation, the value of σ_M will also be large. A small value of the latter quantity indicates a uniform material.

A great value of $r_{M,m}$ and σ_M indicates a possible existence of different groups of stars having different absolute magnitudes, as has been pointed out above. A small value of $r_{M,m}$ suggests that the material ranges within rather close limits of absolute magnitude. Therefore the values above might be regarded as possible, since the presence of different groups has already been shown to be likely. The small $r_{M,m}$ arrived at, when the apparently faint groups of the stars which statistically correspond to the *absolutely* faint groups were excluded, is also in favour of this view. The dispersion σ_M derived from this value of $r_{M,m}$ agrees with that obtained for groups of stars with narrow limits of absolute magnitude, e. g. stars of spectral class B and A. Thus GYLLENBERG³ has found for stars of spectral class A0—F5:

$$\sigma_M = 0^m88 \pm 0^m42. \quad (138)$$

³ Lund Medd Ser I No. 142 (1936).

The dispersion σ_M , derived directly from the compiled list of spectroscopic magnitudes of irregular variables, has the high figure of:

$$\sigma_M = 1^m78 \pm 0^m23. \quad (139)$$

Investigation of the ellipse of correlation M, m .

It is of interest to study the appearance of the ellipse of correlation between the absolute and apparent magnitudes [See Formula (49)]:

$$\frac{1}{2(1 - r_{M,m}^2)} \left[\frac{(M - M_0)^2}{\sigma_M^2} - 2r_{M,m} \frac{(M - M_0)(m - m_0)}{\sigma_M \sigma_m} + \frac{(m - m_0)^2}{\sigma_m^2} \right] = \text{const.} \quad (140)$$

Here $r_{M,m}$ is the coefficient of correlation.

Derivating this expression with regard to the two variates m and M , we obtain two linear equations, representing the two lines of regression:

$$\begin{aligned} f'_m = 0: (1) \quad m &= m_0 + M - M_0, \\ f'_M = 0: (2) \quad M &= M_0 + \frac{\sigma_M^2}{\sigma_m^2} (m - m_0). \end{aligned} \quad (141)$$

The regression-line (1) gives the most probable values of m , when M is known. The regression-line (2) gives the most probable values of M , when m is known.

For the two lines of regression, we observe that the tangent of the angle of inclination of the line (1) is:

$$\text{tg } v_1 = 1. \quad (142)$$

The line thus makes an angle of 45° with the m -axis.

The tangent of the angle of inclination of the regression-line (2) is:

$$\text{tg } v_2 = \frac{\sigma_M^2}{\sigma_m^2}. \quad (143)$$

The centre of the ellipse evidently has the coordinates:

$$\begin{aligned} m &= m_0, \\ M &= M_0. \end{aligned} \quad (144)$$

Then it is easy to construct the ellipse or family of ellipses, when the following quantities are known:

- 1) the lengths of the axes,
- 2) the tangent of the angle of inclination of the diameters.

The reduced equation, when origin coincides with the centre of the ellipse, is written as follows:

$$\sigma_M^2 m^2 + \sigma_m^2 M^2 - 2\sigma_M^2 m M + 2\sigma_M^2 (\sigma_M^2 - \sigma_m^2) = 0. \quad (145)$$

The next procedure is to turn the system of coordinates so that the axes of the ellipse coincide with the axes of the system of coordinates. After this transformation the following form of the equation is obtained:

$$\frac{m^2}{\sigma_m^2 + \sigma_M^2 + \sqrt{\sigma_m^4 + 5\sigma_M^4 - 2\sigma_m^2\sigma_M^2}} + \frac{M^2}{\sigma_m^2 + \sigma_M^2 - \sqrt{\sigma_m^4 + 5\sigma_M^4 - 2\sigma_m^2\sigma_M^2}} = \text{const.} \quad (146)$$

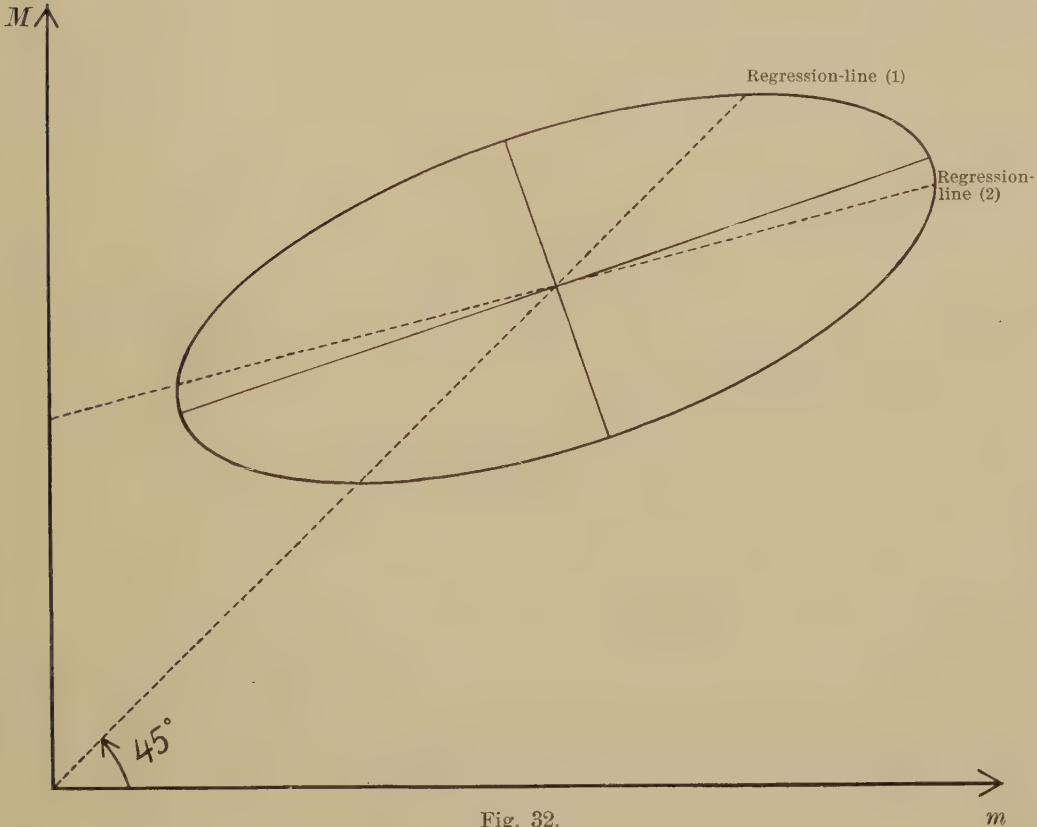


Fig. 32.

The semi-axes are then expressed as follows:

The m -axis:
$$\sqrt{\sigma_m^2 + \sigma_M^2 + \sqrt{\sigma_m^4 + 5\sigma_M^4 - 2\sigma_m^2\sigma_M^2}}, \quad (147)$$

The M -axis:
$$\sqrt{\sigma_m^2 + \sigma_M^2 - \sqrt{\sigma_m^4 + 5\sigma_M^4 - 2\sigma_m^2\sigma_M^2}}.$$

The tangent of the angle φ between the major axis of the ellipse and the m -axis is derived from the following expression:

$$\text{tg } 2\varphi = \frac{2r_{11}}{r_{20} - r_{02}}. \quad (148)$$

Hence we have:

$$\operatorname{tg} 2\varphi = \frac{2\sigma_M^2}{\sigma_m^2 - \sigma_M^2}. \quad (149)$$

In order to construct the ellipse more accurately we may determine the four points of intersection between the ellipse and the regression-lines.

For the first regression-line we then obtain:

$$m = m_0 \pm \sigma_M \sqrt{2}, \quad M = M_0 \pm \sigma_M \sqrt{2}. \quad (150)$$

For the second regression-line we find:

$$m = m_0 \pm \sigma_m \sqrt{2}, \quad M = M_0 \pm \frac{\sigma_M}{\sigma_m} \sqrt{2}. \quad (151)$$

By means of the above relation we are able to construct the ellipse expressing the correlation between M and m .

To simplify a graphical construction, the following values have been assumed:

$$\sigma_m = 2; \quad \sigma_M = 1.$$

Correlation between absolute magnitude and spectrum.

It is known from the investigations in Chapter 3 that there exists a small correlation between apparent magnitude and spectrum. The numerical value of this coefficient of correlation is small and on the limit of reality.

We have:
$$r_{m,s} = +0.104 \pm 0.053. \quad (152)$$

Since the correlation between absolute and apparent magnitude is also positive, it should be positive between absolute magnitude and spectral index.

From the already known values of the mentioned coefficients of correlation, $r_{M,m}$ and $r_{m,s}$, it will be possible to calculate the value of the coefficient of correlation $r_{M,s}$.

According to the definition we have:

$$r_{M,s} = \frac{r_{M,m} r_{m,s}}{\sigma_M \sigma_s}. \quad (153)$$

where:

$$r_{11}(Ms) = \frac{1}{N} \sum (M - M_0)(s - s_0). \quad (154)$$

Further we have:

$$r_{M,m} = \frac{r_{11}(Mm)}{\sigma_M \sigma_m}, \quad r_{m,s} = \frac{r_{11}(ms)}{\sigma_m \sigma_s}, \quad (155)$$

where:

$$\begin{aligned} r_{11}(Mm) &= \frac{1}{N_1} \sum (M - M_0)(m - m_0), \\ r_{11}(ms) &= \frac{1}{N_2} \sum (m - m_0)(s - s_0). \end{aligned} \quad (156)$$

We will assume: $N_1 = N_2 = N$.

Then we obtain, if we observe that:

$$\begin{aligned} \frac{1}{N} \sum (m - m_0)^2 &= \sigma_m^2, \\ \frac{1}{N} \sum (M - M_0)(s - s_0) &= r_{11}(Ms), \end{aligned} \quad (157)$$

$$r_{M,s} = r_{M,m} r_{m,s}. \quad (158)$$

Adopting the values:

$$r_{M,m} = +0.82, \quad r_{m,s} = +0.104,$$

we obtain the following coefficient of correlation:

$$r_{M,s} = +0.085. \quad (159)$$

This figure must represent an upper limit, since the highest value obtained for $r_{M,m}$ has been used. Hence there appears a small correlation between absolute magnitude and spectrum. The line of regression is thus inclined at a very small angle towards the spectrum-axis.

We have attempted to determine one of the regression-lines between absolute magnitude and spectrum, viz. that for given spectrum. In doing this the apex-solution was applied to the proper motions, with the stars grouped according to

spectrum. For this purpose all stars were included, except α Oph and ϱ Per. Table 26 shows the results, obtained from the non-reduced proper motions with apex regarded as unknown.

TABLE 26.
Apex unknown. Non-reduced proper motions. All stars.

Sp.	$\bar{S}$	$\bar{m}_{\max}$	N	$\vartheta_1 S$	r (parsecs)					M_m				
					$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$	$\sigma_M=0$	$\sigma_M=0.5$	$\sigma_M=1.0$	$\sigma_M=1.5$	$\sigma_M=2.0$
F	F7	8.68	4	—	—	—	—	—	—	—	—	—	—	—
G	G6	8.00	11	.0233	187	197	230	301	436	+ 1.64	+ 1.58	+ 1.41	+ 1.12	+ 0.72
K	K3	8.40	21	.0216	202	212	248	325	471	+ 1.88	+ 1.82	+ 1.65	+ 1.36	+ 0.96
M0—M4	M2	8.94	72	.0121	359	379	442	579	839	+ 1.16	+ 1.10	+ 0.93	+ 0.64	+ 0.24
M5—M9	M7	8.93	51	.0204	213	225	262	343	498	+ 2.28	+ 2.22	+ 2.05	+ 1.76	+ 1.36
N	N3	9.30	50	.0063	685	723	843	1104	1601	+ 0.12	+ 0.06	— 0.11	— 0.40	— 0.80

The table shows a faint indication of a positive correlation. The stars of spectral class N fall outside, with a brighter magnitude than the other groups of stars. These stars might therefore be assumed to represent the most luminous part of the irregular variables.

In his investigations on variable stars of different spectral classes PARENAGO⁴ found the absolute magnitude to be independent of spectrum. This is not wholly confirmed by the present investigation, although the correlation obtained is very low.

The value of the absolute magnitude for different spectral classes may be obtained from the galactic characteristics, as has been discussed in Chapter 6. The values of $|10^{0.2m} \sin b|$ have been formed for each spectral class.

Now we have:

$$M = 5 \log |z| + \text{const.} \quad (160)$$

Hence we obtain the relative values of M . In order to compare them with those obtained above, the same spectral intervals have been used.

TABLE 27.

Sp.	$M_{\max}$ (from $ z $)
G	1.49 + const.
K	0.72 "
M0—M4	1.57 "
M5—M9	1.74 "
N	0.25 "

⁴ RAJ XI.1 (1934).

From this table we conclude that the absolute magnitude is practically constant for all spectral classes except possibly for class N. By other methods too these stars are found to be considerably brighter than those of the other spectral classes.

Correlation between absolute magnitude and distance.

Figure 33 illustrates the connexion between the absolute magnitudes M_m for the seven groups of apparent magnitudes (Table 22) and the corresponding mean distances, derived for each group from the parallactic motion. The relation is essentially of the form that would appear from the correlation between apparent

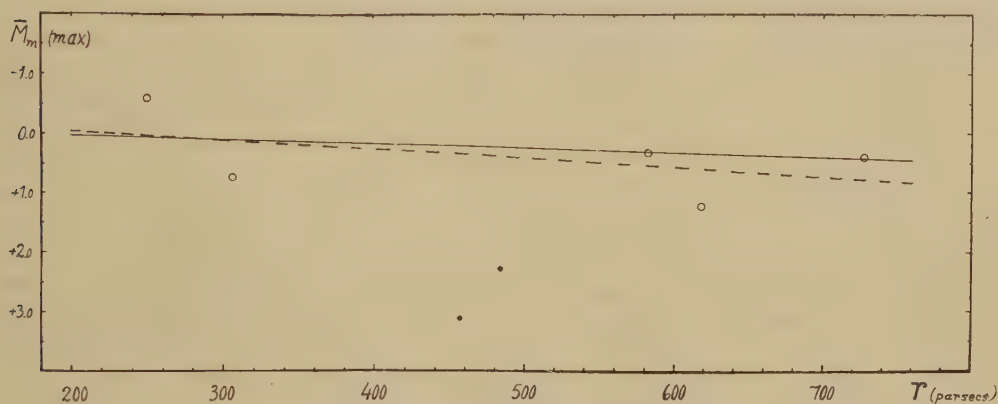


Fig. 33. Correlation between the absolute *maximum* magnitudes, obtained from the proper motions, and the distance.

and absolute magnitudes, i. e. a continuous decrease in the absolute brightness for increasing distances, or more exactly: decreasing apparent brightness. But it is out of the question that the correlation referred to should be exclusively responsible for this effect, since the trend of the figures in that case indicates a coefficient of correlation very near unity.

If real, such a value of $r_{M,m}$ would correspond to a very peculiar distribution of the stars in space. We consider the relation:

$$\sigma_y^2 = \sigma_m^2 - \sigma_M^2, \quad (161)$$

and observe that it involves that $\sigma_M \leq \sigma_m$, which fact is also conspicuous from the condition:

$$r_{M,m} \leq 1. \quad (162)$$

In the case that σ_M is numerically of the same order of magnitude as σ_m we conclude that σ_y must be very small, which accordingly implies that the stars distributed in the Milky Way form a ring, i. e. that they are very little scattered

around their mean distances indicated by the parameter y_0 , defined in Chapter 10, p. 84.

The systematic deviations in the values of M are explained in a more plausible way as an effect of light absorption in the Milky Way regions. Being among the absolutely very brightest star-groups, the irregular variables undoubtedly form a very suitable material for the determination of the absorption in space by means of statistical methods. It is true that the absorption-effect in this case, as in parallel cases, has an effect on the calculated absolute magnitudes similar to that of the correlation between M and m , but these two effects may apparently be separated, when we take into account that one of them is expressed in an arithmetical series and the other in a geometrical one. On the other hand the order of magnitude of the absorption effect is so predominant at the large distances referred to here that, in a first approximation at least, the correlation-effect may be neglected.

From the form of the regression between absolute and apparent magnitude we get:

$$dM' = r_{M,m}^2 dm. \quad (163)$$

Now considering that:

$$r = R_0 10^{0.2m}, \quad (164)$$

(as unity of distance 10 parsecs is adopted) we get:

$$\frac{dM'}{dr} = 5 \text{ Mod } r_{M,m}^2 \frac{1}{r}. \quad (165)$$

This is an expression for the effect depending on the correlation between M and m . To this we have to add the effect of the light-absorption. When it is assumed that the absorption is constant and expressed by α magnitudes per 1000 parsecs, we have:

$$\frac{dM''}{dr} = \alpha. \quad (166)$$

Hence the total decrease in the absolute magnitudes as derived for stars at the mean distance r , is expressed by:

$$\frac{dM}{dr} = \alpha + 5 \text{ Mod } r_{M,m}^2 \frac{1}{r}. \quad (167)$$

In Table 27 the effect $\frac{dM'}{dr}$ has been calculated for six different values of $r_{M,m}$.

When the calculations are referred to the mean absolute magnitudes M_m relating to the corresponding apparent magnitude m of our total material, and if further the mean distance is denoted by r_0 , we have:

$$\frac{dM}{dr} = \alpha + 5 \text{ Mod } r_{M,m}^2 \left(\frac{1}{r} - \frac{1}{r_0} \right). \quad (168)$$

From the points in Fig. 33 a graphical derivation of the quotient $\frac{dM}{dr}$ or α' was made. The last two points of the list were disregarded. The remaining five points gave as result an effect in M of:

$$1^{\text{m}}70 \text{ per } 1000 \text{ parsecs.}$$

Here the two sets of points were referred to their centre of gravitation.

Leaving out point V , on account of its apparent uncertainty, the four points render as result a decrease in absolute magnitude with distance of

$$0^{\text{m}}70 \text{ per } 1000 \text{ parsecs.}$$

The effect also appears from a treatment of the series of magnitudes in Table 22 by the method of least squares.

Let M_0 denote the unaffected absolute magnitude. Then the observed magnitude M has the following expression:

$$M = M_0 + \frac{r}{1000} \alpha', \quad (169)$$

where α' is the effect in absolute magnitude with distance r caused by absorption. The distance is expressed in parsecs. From this equation we obtain the following normal equations:

$$\begin{cases} [M_i] = N M_0 + \alpha' [r_i], \\ [M_i r_i] = M_0 [r_i] + \alpha' [r_i^2]. \end{cases} \quad (170)$$

Here N is the number of equations.

From this system the quantities α' and M_0 have been derived.

Solution 1: — Groups I—VII from Table 22 are taken together, equal weights.

We obtain:

$$\alpha' = + 0.483, \quad M_0 = + 0.85. \quad (171)$$

Solution 2: — Groups VI—VII were excluded and equal weights assigned.

We find:

$$\alpha' = + 0.751, \quad M_0 = + 0.064. \quad (172)$$

Since we have determined the coefficient of correlation $r_{M,m}$ earlier in this paper, we may use the above deductions to find the coefficient of absorption α .

We then consider:

$$\frac{dM}{dr} = \alpha + 5 \text{ Mod } r_{M,m}^2 \left(\frac{1}{r} - \frac{1}{r_0} \right), \quad (173)$$

where the unit of distance is 1000 parsecs. Then α defines the coefficient of absorption. We have to put the mean distance r_0 equal to 527 parsecs, i. e. 52.7 decaparsecs.

TABLE 28.
Effect of distance on M through the correlation between M and m .

r deca- parsecs	$\frac{1}{r}$	$\frac{dM'}{dr}$ $r_{M,m}=0.30$	$\frac{dM'}{dr}$ $r_{M,m}=0.40$	$\frac{dM'}{dr}$ $r_{M,m}=0.50$	$\frac{dM'}{dr}$ $r_{M,m}=0.60$	$\frac{dM'}{dr}$ $r_{M,m}=0.70$	$\frac{dM'}{dr}$ $r_{M,m}=0.80$
1	1.000	+ 0.195	+ 0.347	+ 0.543	+ 0.782	+ 1.064	+ 1.390
5	0.200	.039	.069	.109	.156	.213	.278
10	.100	.020	.035	.054	.078	.106	.139
15	.067	.013	.023	.036	.053	.071	.093
20	.050	.010	.017	.027	.039	.053	.069
25	.040	.008	.014	.022	.031	.043	.056
30	.033	.007	.011	.018	.026	.035	.046
35	.029	.006	.010	.016	.023	.031	.040
40	.025	.005	.009	.013	.020	.026	.035
45	.022	.004	.008	.012	.017	.023	.030
50	.020	.004	.007	.011	.016	.021	.028
55	.018	.003	.006	.010	.014	.019	.025
60	.017	.003	.006	.009	.013	.018	.023
65	.015	.003	.005	.008	.012	.016	.021
70	.014	.003	.005	.007	.011	.015	.020
75	.013	.003	.004	.007	.010	.014	.018
80	.012	.002	.004	.007	.010	.013	.017
85	.012	.002	.004	.007	.010	.013	.017
90	.011	.002	.004	.006	.009	.012	.015
95	.011	.002	.004	.006	.009	.012	.015
100	.010	.002	.003	.005	.008	.010	.014

From Table 28 it is seen that the second term in maximum is equal to 0^m03. The influence of a variation in $r_{M,m}$ upon this term is evidently unessential.

Hence we obtain the following values of the effect of absorption:

Method	α'	α
1	+ 0 ^m 75	+ 0 ^m 72
2	+ 0.48	+ 0.45
3	+ 0.70	+ 0.67
(4)	+ 1.70	+ 1.67)

The figures in the last line must be considered as less accurate.

From the values found above it appears probable that the coefficient of absorption is essentially smaller than 1^m per 1000 parsecs, a fact that is also deduced from the investigations of the galactic z' -coordinates.

From the first three lines in the above table we form a mean value of α . We find:

$$\alpha = + 0^m613. \quad (174)$$

Perhaps the simplest way to find the light-absorption in the Milky Way is to enter the absorption-coefficient into the original solution of the distance and regard the stars in a direction normal to the galactic plane as free from absorption.

Starting from the definition of the distance:

$$r = R 10^{0.2m},$$

where:

$$R = 10^{-0.2M},$$

we obtain the following expressions, where it is assumed that the stars in the Milky Way are ΔM magnitudes fainter than those outside on account of the absorption:

$$\begin{aligned} (1) \quad r_1 &= 10^{0.2m} 10^{-0.2M}, \\ (2) \quad r_2 &= 10^{0.2m} 10^{-0.2(M+\Delta M)}. \end{aligned} \quad (175)$$

Observing that

$$10^{-0.2\Delta M} = e^{-b\Delta M} = 1 - b\Delta M, \quad (176)$$

(2) can be written in the following form:

$$r_2 = R 10^{0.2m} (1 - b\Delta M). \quad (177)$$

We may further put:

$$b\Delta M = x.$$

Then x can be solved from the whole material.

When a sufficiently large material is at disposal, another method to determine the absorption may be used. The absorption will evidently vary, depending on the position of the stars in relation to the direction to the galactic centre. The absorption will reach its maximum value in the direction towards the centre, and its minimum value in the opposite direction. Between these two limits the absorption will vary from a maximum value to a minimum value. The total effect can be written as a sum of two terms: the constant minimum absorption and the continuously varying value of the absorption, which depends on the position of the stars.

Let the star be situated λ degrees from the centre; then the effect will vary proportional to $\cos \frac{1}{2} \lambda$. Hence the total absorption-effect α can be written:

$$\alpha = \alpha_1 + \alpha_0 \cos \frac{1}{2} \lambda. \quad (178)$$

The maximum value of α occurs when $\lambda = 0$. Then $\cos \frac{1}{2} \lambda = +1$, and

$$\alpha = \alpha_1 + \alpha_0. \quad (179)$$

The minimum value occurs when $\lambda = 180^\circ$, or when $\cos \frac{1}{2} \lambda = 0$.

$$\text{Then} \quad \alpha = \alpha_1. \quad (180)$$

The last two methods have only been briefly explained, since the present material has not permitted a numerical application.

The absolute magnitude of the stars constituting the secondary maximum of the frequency-curve of the apparent magnitudes.

Spectroscopic magnitudes.

In order to find an explanation of the presence of the secondary maximum of the frequency-curve of the apparent magnitudes, the stars belonging to this group were investigated both individually and as a group with regard to their absolute magnitudes.

For all stars of the group except two it has been possible to determine the absolute spectroscopic magnitude and spectroscopic parallax.

From Table 29 it is seen that most of the stars have very bright absolute magnitude. This quantity, fundamentally a criterion of the mass of the stars, classifies the irregular variables of frequency-maximum *A* as giants. The variables below apparent galactic latitude $\pm 30^\circ$ are in fact also found absolutely closer to the galactic plane than those above these latitudes. A few of the stars are evidently fairly adjacent to us. However, from the data given in the table it cannot be considered to be proved that these stars are members of a local star-cloud, as might be supposed possible.

This group of irregular variables has also been studied statistically in the following paragraph.

TABLE 29.

Irregular variables belonging to frequency-maximum *A* of the apparent magnitudes.

Var.	$m_{\max}$ [phot.]	$m_{\min}$ [phot.]	Sp.	Period	l	b	$10^{0.2m_{\min}} \sin b$	M_{sp}	π_{sp}	z [parsecs]	r [parsecs]
α Ori	1 ^m 8	2 ^m 9	M2		168°	— 8°	— 0.6	— 4 ^m 0	.012	— 11.5	83
α Cas	3.1	3.7	G8		89	— 6	— 0.5	— 1.1	—	—	—
β Peg	4.1	4.6	M3	34 ^d + 1p	64	— 29	— 3.9	— 0.5	—	—	—
κ Oph	4.1	5.0	K0		356	+ 28	+ 4.7	+ 1.2	.037	+ 14.5	31
ϱ Per	4.9	5.8	Mb	33 ^d ; 910 ^d	118	— 16	— 3.9	— 0.5	.012	— 22.9	83
δ Ser	4.9	5.6	A0p	irr	358	+ 4	+ 0.9	+ 0.9	.008	+ 8.7	125
η Gem	4.9	5.9	M2	235 ^d 58	157	+ 3	+ 0.8	— 1.0	.013	+ 4.0	77
α Her	4.8	5.6	M5	89 ^d 2; $\sim 2000^d$	3	+ 26	+ 5.7	— 2.3	.009	+ 48.6	111
AE Aur	5.0	5.9	B0p	40 ^d — 50 ^d	140	— 1	— 0.3	— 2.4	.003	— 5.7	333
ϱ Cas	5.4	6.1	cG5		83	— 4	— 1.2	— 4.5	.003	— 23.3	333
CI Ori	5.1	6.2	K5		172	— 17	— 5.0	— 0.2	.010	— 29.2	100
R Lyr	5.7	6.2	M5	45 ^d — 75 ^d } 1500 ^d 15 — 60	41	+ 17	+ 5.0	— 1.3	.008	+ 36.5	125
VV Cep	6.1	6.8	M2ep	60 ^d ; 900 ^d	72	+ 7	+ 2.8	— 2.5	.003	+ 40.6	333
μ Cep	5.7	6.5	M2		68	+ 4	+ 1.4	— 4.0	.002	+ 35.0	500
CK Ori	6.7	7.3	K0		167	— 15	— 7.5	—	—	—	—
R Apo	6.5	7.7	K2		277	— 16	— 9.7	—	—	—	—
g Her	6.1	7.3	Mbp	(50 ^d — 90 ^d) + 1p	33	+ 43	+ 19.8	— 0.7	.006	+ 113.9	167
RR Ari	6.8	7.3	K0		111	— 36	— 17.1	+ 0.9	.007	— 84.1	143
T Cet	6.9	7.7	M5	158 ^d 8 hr	51	— 81	— 34.6	— 0.6	.006	— 165.0	167

Mean absolute magnitude on basis of the proper motions.

To deduce the mean parallax and mean absolute magnitude, the proper motions of this group were treated with the methods of the apex-solution. In so doing the two stars α Oph and ϱ Per were excluded on account of their extraordinarily large proper motions. Thus 17 variables were used. The solution was made using the coordinates of the Standard Apex. Only the minimum magnitudes were used, since the material was selected by means of the minimum-magnitude curve.

The mean distances and the mean absolute magnitudes were corrected in the usual way for the dispersion in absolute magnitude.

Then the following values are obtained:

I. Non-reduced proper motions: $\vartheta_1 S = .0216$.

$$\begin{array}{ll} [\sigma_M = 1^m0] & [\sigma_M = 1^m5] \\ r = 247 \text{ parsecs,} & r = 324 \text{ parsecs,} \\ M_{\min} = -0.68, & M_{\min} = -0.97. \end{array}$$

II. Reduced proper motions:

$$\begin{array}{ll} \Theta_1 S = .5023 & R = 8.66 e^{b^2 \sigma_M^2} \text{ parsecs} \\ [\sigma'_M = 1^m0] & [\sigma_M = 1^m5] \\ M_{\min} = +0.08, & M_{\min} = -0.21, \\ R = 10.6 \text{ parsecs,} & R = 13.9 \text{ parsecs.} \end{array}$$

However, since the stars belong to the galactic region, a correction for the absorption should be applied. $r = 300$ parsecs approximately might be accepted here.

Investigations of the absorption show this effect with great probability to be less than 1^m per 1000 parsecs. The mentioned correction applied for this effect is a maximum value. The lower numerical value of the coefficient, 0^m5 per 1000 parsecs, has also been taken into account. In the two cases we obtain respectively the corrections:

$$\begin{array}{l} \Delta M_1 = -0^m30, \\ \Delta M_2 = -0.15. \end{array}$$

The values of $M_{\max}$ have been found by assuming the amplitude of variation equal to 1^m50 .

Then the following values of the corrected absolute magnitude are obtained for the frequency-maximum A :

TABLE 30.

Abs.	$\bar{M}_{\min}$		$\bar{M}_{\max}$	
	$\sigma_M = 1.0$	$\sigma_M = 1.5$	$\sigma_M = 1.0$	$\sigma_M = 1.5$
0^m5	-0^m83	-1^m12	-2^m33	-2^m62
1.0	-0.98	-1.27	-2.48	-2.77

It consequently appears from the above discussion that the small frequency-maximum of apparently bright stars consists of irregular variables situated at very low galactic latitudes, with brighter absolute magnitudes than the main bulk of the stars. Therefore the probability is great that this group represents a super-giant class of irregularly variable stars.

Comparison with the long-period variables.

The stars belonging to the bright frequency-maximum *A* of the long-period variables have been treated in the same way as those of the irregular variables. From the available proper motions, embracing 29 stars, we have calculated the mean parallax and mean absolute magnitude for the two groups of stars in the galactic zone $\pm 30^\circ$ and outside this zone. The mean of the apparent magnitudes proved to be the same for the two groups. With regard to spectra and amplitudes there are no differences. The same fact as was observed for the galactic and non-galactic irregular variables appears here: the stars in the galactic zone have a brighter absolute magnitude than those outside the zone. (See Table 23.)

Long-period variables. Frequency maximum *A*.

Apex known = Standard Apex.

I.	$b \leq \pm 30^\circ;$	$N = 12;$	$\bar{m}_{\min} = 11.68.$	
	Non-reduced proper motions:		Reduced proper motions:	
	$\vartheta_1 S = -0.00445.$		$\Theta_1 S = -1.7249.$	
	$[\sigma_M = 1^M0]$	$[\sigma_M = 1^M5]$	$[\sigma_M = 1^M0]$	$[\sigma_M = 1^M5]$
	$r = 1202 \text{ par},$	$r = 1574 \text{ par},$	$R = 3.10 \text{ par},$	$R = 4.06 \text{ par},$
	$M_{\min} = +1.50,$	$M_{\min} = +1.21,$	$M_{\min} = +2.76,$	$M_{\min} = +2.47.$
II.	$b > \pm 30^\circ;$	$N = 17;$	$\bar{m}_{\min} = 11.26.$	
	Non-reduced proper motions:		Reduced proper motions:	
	$\vartheta_1 S = -0.0222.$		$\Theta_1 S = -3.3826.$	
	$[\sigma_M = 1^M0]$	$[\sigma_M = 1^M5]$	$[\sigma_M = 1^M0]$	$[\sigma_M = 1^M5]$
	$r = 241 \text{ par},$	$r = 316 \text{ par},$	$R = 1.58 \text{ par},$	$R = 2.07 \text{ par},$
	$M_{\min} = +4.57,$	$M_{\min} = +4.28,$	$M_{\min} = +4.22,$	$M_{\min} = +3.93.$

The distances for galactic and non-galactic stars show a remarkable similarity for the two groups of stars.

Applying the above corrections for absorption, we obtain the following values of *M*.

Corrections for absorption.

α	ΔM	
	$r = 1200 \text{ par}$	$r = 1570 \text{ par}$
0.5	-0.60	-0.78
1.0	-1.20	-1.57

$$M_{\min} (b \leq \pm 30^\circ)$$

α	Non-reduced proper motions		Reduced proper motions	
	$\sigma_M = 1.0$	$\sigma_M = 1.5$	$\sigma_M = 1.0$	$\sigma_M = 1.5$
0 ^m 5	+ 0.90	+ 0.43	+ 2.16	+ 1.69
1.0	+ 0.30	— 0.36	+ 1.56	+ 0.90

The mean uncorrected magnitudes for all 29 stars are given below.

Uncorrected mean value: $N = 29$.

$$M_{\min} = + 3.26 \quad (\sigma_M = 1.0),$$

$$M_{\min} = + 2.97 \quad (\sigma_M = 1.5).$$

These values agree with the mean magnitude of all long-period variables of + 3^m.36, found by GYLLENBERG.⁵

Further discussion of the absolute magnitudes.

In the discussion of the distribution of the apparent magnitudes it was pointed out that the difference between the maxima of three curves (irregular, long-period and S-variables) is about 5^m, as derived from the curve [cf. Table 31]. We will now discuss this fact a little more closely.

TABLE 31.

The apparent magnitudes of the secondary maximum and the main maximum of the frequency-curves of different classes of variables.

Class	Sec. max		Main max	Main max— sec. max		Irr—Lp (sec. max)		Irr—Se (sec. max)	Lp—Se (sec. max)		Main max	
	1	2		1	2	1	2		1	2	Irr—	Lp—
Irr	5.6		11.5	5.9								
Lp	11.4	12.6		(4.0)		— 5.8			— 0.3			
b < 30°	10.2	11.7	15.4	5.2	3.7		— 6.1		— 1.5	0.0	— 3.9	
b < 30°	11.7			3.7		— 6.1			0.0			
All	11.7		15.8	4.1				— 6.1			— 4.3	— 0.8
Se			11.0								+ 0.5	+ 4.4
N												

If stars of the separate frequency-maxima of the irregular variables had the same mean distance from us, the difference in apparent magnitude would equal the difference in absolute magnitude. Since $m = M + 5 \log r$, we have for the two groups:

$$\begin{aligned} \text{For group } A: & \quad m_1 = M_1 + y_1, \\ \text{» » } B: & \quad m_2 = M_2 + y_2. \end{aligned} \tag{181}$$

⁵ Lund Medd Ser II No. 54 (1930).

Here m_1 is assumed to be brighter than m_2 .

We have:

$$m_1 - m_2 = (M_1 - M_2) + (y_1 - y_2). \quad (182)$$

If $y_1 = y_2$, then $\overline{5 \log r}$ is the same for both groups, i. e. the stars have the same mean distance. Hence:

$$M_1 - M_2 = m_1 - m_2. \quad (183)$$

For $y_1 \neq y_2$ we can write:

$$m_1 - m_2 = M_1 - M_2 + a, \quad (184)$$

where a is a constant and equal to $y_1 - y_2$.

When solving the absolute magnitude M_1 we find, if M_2 is regarded as known:

$$M_1 = M_2 + (m_1 - m_2) - a. \quad (185)$$

We have the following data:

Minimum magnitudes.

Apparent magnitudes				Absolute magnitudes			
A	B	Method	Diff.	Abs.	A $\sigma_M = 1^m5$	B $\sigma_M = 1^m5$	Diff.
6.1	11 ^m 9 9.7	computed mean values mean value of stars with proper motions	5 ^m 8 3.7	0 ^m 5 1.0	— 1 ^m 1 — 1.3	+ 2 ^m 4	3 ^m 5 3.7

Introducing these values into equation (185) we find, when considering only the stars with known proper motions:

$$\begin{aligned} (1) \quad & -3.7 = -3.5 + a \quad (\text{absorption} = 0^m5 \text{ per } 1000 \text{ parsecs}), \\ (2) \quad & -3.7 = -3.7 + a \quad (\text{absorption} = 1.0 \gg 1000 \gg). \end{aligned} \quad (186)$$

Case (1) gives a negative value of a , thus indicating a greater distance for group B than for group A . Case (2) would suggest the same mean distance for the two groups of stars.

Now from the proper motions we actually obtain a smaller distance for group A than for the rest of the irregular variables, thus indicating the smaller value of the absorption to be more in accordance with the observations. However, a change in the used value of σ_M would have a parallel effect. Therefore it is not possible to establish in this way the exact amount of the absorption.

The above reasoning does, however, prove the existence of a real difference between groups A and B with regard to absolute as well as to apparent magnitude.

Concerning the difference between the frequency-maximum A and the total mean magnitude, we also have to make a comparison between the absolute magnitudes. The correction needed to reduce the magnitude M_m , obtained from the proper motions, to the value M_0 , should in this case be equal to $+1.4$.

$$\begin{aligned} \text{Then:} \quad (1) \quad & M_1 - M_2 = -4.9, \\ (2) \quad & M_1 - M_2 = -5.0. \end{aligned} \tag{187}$$

Here a has a larger negative value, since $m_1 - m_2 = -5.8$, indicating a value of r_2 that is greater than r_1 , as also is actually found.

Thus it should be concluded that the frequency-maximum A comes from stars which are absolutely brighter than the stars forming maximum B , and the latter are more remote from us than the former.

It is of interest to note that the magnitude-difference of 5^m has also been observed for other classes of stars. Thus there was found by G. STRÖMBERG, who investigated the absolute magnitudes of super-giants and giants,⁶ a difference between their mean magnitudes of

$$M_c - M = -5.8.$$

This coincidence might constitute further evidence that the discussed groups of irregular variables are super-giants. The principal frequency-maximum, embracing the main bulk of the stars, would consequently consist of ordinary giants, as has always been supposed.

From the above considerations it seems probable that the stars group themselves around certain magnitude-lines in the Russel diagram which differ by about 5 magnitudes.

GYLLENBERG referred to the possibility of the existence of several different layers in the Russell diagram in connexion with the long-period variables.⁷ The top-points of the individual frequency-curves are clearly indicated in the diagram of period-apparent magnitude. But here the difference between the maxima is about 2.3^m . However, twice this value gives nearly 5^m . There might, of course, be one more frequency-maximum, concealed in the present treatment, among the irregular variables and the super-giants.

When we consider the galactic distribution of the irregular variables, the assumption of a difference in absolute magnitude of about 2.3^m becomes probable.

At present these considerations must remain hypothetical. However, the suggestion is of great interest, and deserves to be mentioned.

Further, several frequency-maxima might exist among the fainter magnitudes. The present curve of distribution in fact suggests their presence. But owing to the incompleteness of the material for these magnitudes, they cannot be established with certainty.

⁶ Mt Wilson Contr 293 (1925).

⁷ Lund Medd Ser II No. 54 (1930).

CHAPTER 12

Irregular variables in the Magellanic Clouds and in other galaxies than ours

Variable stars are not limited to our own galactic system. A number of such stars have also been identified in other galaxies.

The observation of variable stars in other galaxies is of the greatest importance for many problems in stellar astronomy. Since the stars in other systems can be assumed to be situated at the same distance from us, the observed apparent magnitudes of the variables will immediately give the absolute magnitudes, provided the distance of the system is known. Inversely, when the absolute magnitude can be determined independently, variable stars may suitably be used as distance indicators.¹

Irregular variables have been observed in several other galaxies, though in comparatively small number. In some systems only very few have as yet been identified, in others a good many have been found and observed.

In the two Magellanic Clouds are found several irregular variables. H. SHAPLEY and Miss JENKA MOHR have examined the variables in these galaxies in several papers. The result has been published in different Harvard Annals and Harvard Circulars. Irregular variables have been recognized, but it has been possible to establish the period for only a few of them. The authors cited have determined a period-luminosity relation based on 1346 variables in the large Cloud.² However, since the largest period amounts to 100^d, only the upper part of the frequency-curve of the irregular variables is represented. The luminosity-curve obtained evidently represents only the ascending branch of the general period-luminosity curve. At a period of about 100^d the curve seems to turn, hence indicating the presence of a maximum point near this period-value. It may be pointed out that the general period-luminosity curve for galactic variables has also a maximum around 100^d, corresponding to the average period of the irregular variables. This

¹ This term has been introduced by LUNDMARK, who has developed a number of methods to make use of different kinds of distance indicators for the determination of the huge distances in Metagalactic Astronomy.

² HA Vol 90 No. 1 (1933).

average period corresponds to an absolute photographic magnitude of about -5 , obtained from the extended period-luminosity curve of the Cepheids in the Magellanic Clouds. However, this magnitude is much brighter than the mean absolute magnitude found from the present solutions of the proper motions of irregular variables in the Milky Way. Even the brightest value found, i.e. that of the irregular variables in the Milky Way region, does not amount to -5^M . Individual absolute magnitudes, however, sometimes do not differ very much from this limit. Examples are ϱ Cas with $M = -4.5$; μ Cep and α Ori with $M = -4.0$. These stars may belong to the absolutely brightest group of irregular variables in our Galaxy, which are few in number, and represented by the frequency-curve of apparent magnitudes in maximum A .

Below is given a list of irregular variables discovered in other galaxies. The absolute magnitudes have either been collected from the cited papers or have been computed in this paper by means of the distance-modulus ($M - m$) given for the object in question from SHAPLEY's, LUNDMARK's and HUBBLE's investigations.

The best investigated objects are of course the two Magellanic Clouds and the Andromeda galaxy.

For the present list of irregular variables in the two Magellanic Clouds the Harvard investigations by SHAPLEY,³ SHAPLEY and Miss MOHR⁴ and Miss HOFFLEIT⁵

TABLE 32 a.

Irregular variables in the Large Magellanic Cloud.

[DORRIT HOFFLEIT; HB 905].

 $m - M = 17.1$.

HV	$m_{\max}$	$m_{\min}$	A	P	Derived		Remarks
					$M_{\max}$	$M_{\min}$	
888	13.2	14.8	1.6		-3.9	-2.3	lp or irr; if lp, per. = 1000 ^d
916	14.0	16.0	2.0		-3.1	-1.1	lp
2602	14.6	15.6	1.0		-2.5	-1.5	appears irr
2677	15.2	16.8	1.6		-1.9	-0.3	apparently irr
2700	14.0	16.0	2.0		-3.1	-1.1	lp or irr?
2740	15.0	16.8	1.8		-2.1	-0.3	HB 833 period spurious; apparently irr
2827	12.5	14.3	1.8		-4.6	-2.8	apparently irr
5891	13.7	14.6	0.9		-3.4	-2.5	irr
5961	13.8	14.6	0.8		-3.3	-2.5	irr? possibly eclipsing var.
5964	13.5	14.3	0.8		-3.6	-2.8	irr?
2447	12.5	14.0	1.5	118.5	-4.6	-3.1	Sp.: K5
900	13.0	14.5	1.5	47.52	-4.1	-2.6	
953	12.3	13.7	1.4	47.7	-4.8	-3.4	
Mean			1.44		-3.46	-2.02	
2822	10.8	11.4	0.6		-6.3	-5.7	Sp.: Mc; possibly irr; too bright for reliable estimates.
S Dor	8.2	9.4	1.2		-8.9		irr; Sp.: Pec

³ HC 255 (1924), 260 (1924), 268 (1924); H. SHAPLEY and HARVIA H. WILSON: HC 271 (1925), 275 (1925), 276 (1925), 278 (1925).

⁴ HA 90.1 (1933).

⁵ HB 882 (1931), 886 (1932), 900 (1935), 905 (1937).

TABLE 32 b.
Irregular variables in the Small Magellanic Cloud.

[DORRIT HOFFLEIT; HB 900].

 $m - M = 17.1$.

Var	$m_{\max}$	$m_{\min}$	m_{med}	A	P	Derived		Remarks
						$M_{\max}$	$M_{\min}$	
HV								
829	<i>12.7</i>	<i>13.5</i>	13.1	0 ^m .8	88 ^d .5	— 4.4	— 3.6	
834	<i>12.30</i>	<i>13.40</i>	12.85	1.1	73.5	— 4.8	— 3.7	
1877	<i>13.8</i>	<i>14.4</i>	14.1	0.6	49.6	— 3.3	— 2.7	
1455	14.3	15.1		0.8	irr?	— 2.8	— 2.0	
1880	14.5	15.2		0.7		— 2.6	— 1.9	short period or irr?
2084	13.6	15.0		1.4		— 3.5	— 2.1	irr or long period
2232	14.2	15.2		1.0		— 2.9	— 1.9	probably irr
Mean				0.90		— 3.47	— 2.56	
1956	12.0	14.2	13.1	2.2	209.	— 5.1	— 2.9	
864	11.	15.		4.	315 ±	— 6.	— 2.	

have mainly been made use of. Miss HOFFLEIT gives a list of magnitudes and periods for variable stars in the two Clouds. From these lists the irregular variables were compiled and the absolute magnitudes deduced by means of the distance-modulus, which is found to equal 17^m.1. In all, 21 variables have been recognized as irregular in the Clouds. In this number is included S Dor, which star is the brightest variable known, with an absolute magnitude of — 8.9.⁶ This variable has been classified by LUNDMARK as a permanent Nova of the P Cygni-class. The limit between these two classes of stars is indefinite.

All of the stars have very bright absolute magnitudes. In many cases the irregular variables are reported by the authors to be the very brightest objects of the system.

If we consider the list of irregular variables in the Magellanic Clouds, we derive the following mean absolute photographic magnitudes (S Dor excluded):

$$M_{\max} = -3.46 \pm 0.20,$$

$$M_{\min} = -2.29 \pm 0.21.$$

From the papers of the above mentioned authors the irregular variables found in the galaxies have been compiled. In some cases the magnitudes at maximum and minimum are given, in other cases the mean magnitude and the amplitude are given. From these data the maximum and minimum magnitudes are computed. If the *absolute* magnitudes are not given in the papers, they have been derived by means of the distance-modulus, and are printed in italics in Table 33.

From the list it is seen that on an average the absolute magnitudes of irregular variables found in other galaxies are brighter than those found in the Magellanic Clouds and the Milky Way.

⁶ HB 814 (1925), HC 271 (1925).

TABLE 33.

Irregular variables in anagalactic objects.

System	$m_{\max}$	$m_{\min}$	A	$M_{\max}$	$M_{\min}$	Remarks
Var						
And = M 31	14.7	15.6	<i>0^m.9</i>	— 7.5		HUBBLE
11	18.6	19.6	<i>1.0</i>	— 3.6 — 3.2	— 2.6 — 2.2	
15	16.7	17.5	<i>0.8</i>	— 5.5 — 5.1	— 4.7 — 4.3	$c < 0^m.2$
19	15.3	16.3	<i>1.0</i>	— 6.9 — 6.5	— 5.9 — 5.5	brightest of all variables indicated period of about 5 years
20	18.0	18.6	<i>0.6</i>	— 4.2 — 3.8	— 3.6 — 3.2	
44	18.7	19.4	<i>0.7</i>	— 3.5 — 3.1	— 2.8 — 2.4	
M 33 2	15.55	17.4	<i>1.9</i>	— 6.55	— 4.7	brightest star in neb. cont. Sp.; suggesting early type; bright H_{β}
1	17.2	18.4	<i>1.2</i>	— 4.9	— 3.7	$c = + 0.5$. 18 ^m .4 seems to be the normal magnitude
Mean			1.01	— 5.32	— 4.00	M 33 has two more irr variables, for which no data are available
M 31:	Var 43 noted as »irr» and »red»; further data missing					
M 31:	$m - M = 22.2$, based on SHAPLEY's value of $m - M$ for the Small Mag. Cloud = 17.55. Revised value for Small Cloud = 17.1. Then $m - M = 21.8$ for M 31. Figures in italics give the corrected values of M .					
M 33:	$m - M = 22.1$ has two more »irr», without further data.					

M 81
 NGC 2403 } stars resembling the bright »irr» in M 31 and M 33. [HUBBLE]
 M 101 }

	$m_{\max}$	$m_{\min}$	P	
NGC 6723	14.0	< 16.0	90 ^d .4	
	12.0	13.5	64.1	
	10.0	< 17.0	347.5	BAILEY [HC 266 (1924)]

NGC 598
 = M 33 Tri Has 4 irr [see table]
 Of the 35 Cepheids 3 have periods > 45^d: 46^d, 55^d, 70^d resp.

[These three variables might really belong to the irregular class.]

Thus: $M_{\max} = -5.32 \pm 0.40$,
 $M_{\min} = -4.00 \pm 0.32$.

The brightest variable in other galaxies has been found in the Andromeda nebula. Its maximum magnitude is -7.5 .

General discussion of the irregular variables in other galaxies than ours.

Comparison with the galactic variables.

Of course we must expect to obtain in our present observations only the brightest variables in other galaxies than ours. Since the apparently brightest stars are probably first observed and investigated, there is great probability that they are stars of high absolute magnitude, on account of the correlation existing between apparent and absolute magnitude.

The very bright stars obtained in certain galaxies are also a natural consequence from the general law of probability. We can easily calculate the number of objects brighter than e. g. three times the dispersion in absolute magnitude, counted from the mean value of the latter. This number depends on the total number of individuals and on the value of the dispersion. Assume this latter to be equal to 1^m and the mean value of the absolute maximum magnitude to be -1^m . Then three times the dispersion corresponds to a limiting magnitude of -4^m . Further, for a number of 10 000 irregular variables, 20 stars might be expected to appear outside this limit, for 1 000 stars only 2. Therefore high magnitudes are always statistically expected.

There is further a peculiar circumstance about the high luminosities, namely that such bright irregular variables have never been observed in our own Galaxy. From the list of individual absolute magnitudes it is found that the brightest is $-4^m.5$. There seems also to be closer similarity in brightness between the irregular variables of our Galaxy and those of the Magellanic Clouds (not including S Dor) than between those of our Galaxy and those of other galaxies.

It is of interest to study the frequency-curve of the absolute magnitudes in other galaxies than ours (Fig. 34).

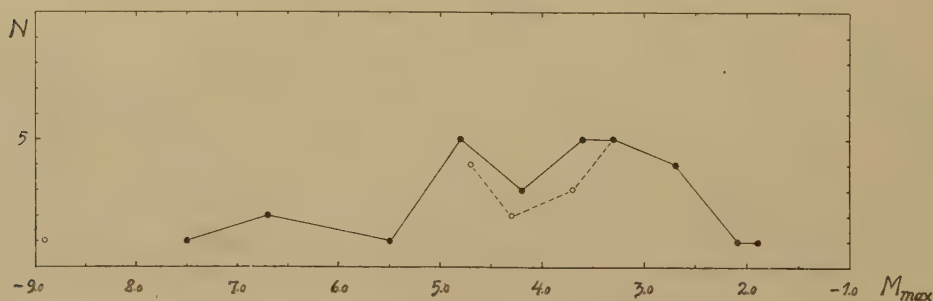


Fig. 34. Distribution of the absolute magnitudes of the irregular variables in other galaxies.
 - - - Absolute magnitudes in the two Magellanic Clouds
 — Absolute magnitudes in all available anagalactic objects
 } At the end the two curves coincide.

This frequency-distribution corresponds to the maximum A in the distribution of apparent magnitudes in our system. The main part of the curve is completely missing.

Fig. 34 gives what corresponds to the first frequency-maximum of the irregular variables in our system. The maximum point for the Magellanic Clouds gives $M = -3.2$. All irregular variables give $M_{\max} = -3.8$ (or about -4.0). The general curve indicates another frequency-maximum for $M = -6.7$. The curve of the Magellanic Clouds rises for the brighter magnitudes, indicating a second maximum for brighter magnitudes.

For the first frequency-maximum in our Galaxy there is obtained from the proper motions: $\bar{M}_{\max} \simeq -2.7$ (corrected value). For the stars in the galactic plane ($b \leq \pm 15^\circ$) there is obtained: $\bar{M}_{\max} = -3.8$ (corrected value).

The distribution of the maximi-magnitudes of irregular variables in the Magellanic Clouds has a maximum at $M_{\max} = -3.25$. At $M = -4.3$ the curve tends upward, thus suggesting the possible presence of another maximum at brighter magnitudes.

The curve representing all magnitudes has three maxima, one at $M = -3.8$, another at $M = -4.8$ and a third at $M = -6.7$. The last group may contain the members of a super-super-giant group of irregular variables, of which no members seem as yet to be observed in the Milky Way.

These magnitudes are considerably brighter than those obtained for the bright galactic irregular variables. The bright variables found in other galaxies are below compared with those of frequency-maximum A of the irregular variables in our Galaxy.

Absolute mean magnitude of the stars of frequency-maximum A .

$M_{\max}$	$M_{\min}$	M e t h o d
-0.72	-1.69	Non-red. p. m.; apex unknown
-3.00		» » apex known
-2.14	-0.97	Red. p. m. »
	-0.21	Non-red. p. m. »
		» »
$\bar{M} = -1.95$	-0.96	

Simple means give the following absolute magnitudes:

$$M_{\max} = -1.95,$$

$$M_{\min} = -0.96.$$

When the correction for the light-absorption is applied, we obtain the following figures, which may represent upper limits in brightness.

$$\begin{aligned} M_{\max} &= -2.77 \quad \left\{ \begin{array}{l} (\sigma_M = 1^{\text{M}}5; \\ \text{absorption} = 1^{\text{M}}0/1000 \text{ parsecs}). \end{array} \right. \\ M_{\min} &= -1.27 \end{aligned}$$

These magnitudes are not considerably fainter than those observed in the Magellanic Clouds. Further, if the individual magnitudes are considered, we observe that the brightest members of our system have the absolute spectroscopic magnitude of -4.5 . Hence there seems to be great probability that the stars of the frequency-maximum A have the same mean luminosities as the bright variables observed in the Magellanic Clouds. These stars are with certainty super-giants.

Another possibility is that the observed apparent magnitudes, and hence the absolute magnitudes, of the stars in our Galaxy are systematically too faint. It cannot be pointed out, at least for the present, where such an error would appear, but the possibility should not be overlooked.

On the other hand the surveys of our Galaxy give a possibility for observing variables having a characteristic »dwarf»-nature.

The catalogue of proper motions

For a detailed discussion of the proper motions of the present catalogue we may refer to Chapter 8. Here we may just add a few explanatory notes concerning the catalogue itself.

The catalogue contains 182 variable stars. A number of these stars belong to other classes of variables than those specially treated in the present paper, though the main part consists of irregular variables. The former stars were observed and their proper motions computed at an earlier stage of the present work, and most of them were included in the author's earlier catalogue of proper motions.¹ The present catalogue includes a revision of the earlier one together with a number of new proper motions. Therefore it was considered justified to include all proper motions of the earlier list in the new catalogue.

The columns of the catalogue are explained as follows:

- Column 1: The current number of the stars in the catalogue.
- Column 2: The name of the variable.
- Column 3: The spectrum of the variable from »Katalog und Ephemeriden veränderlicher Sterne für 1938».
- Column 4: The weighted right-ascension, reduced to the equinox 1900.0 with NEWCOMB's precession-constant and with the proper motion.
- Column 5: } The precession and secular variation for 1900.0, calculated by means
- Column 6: } of NEWCOMB's constant.
- Column 7: The proper motion in right-ascension in seconds of time with its mean error; system Boss PGC.
- Column 8: Epoch of observation in right-ascension.
- Column 9: The number of catalogues used for the determination of the proper motion.
- Column 10: The current number repeated.
- Column 11: The weighted declination, reduced to 1900.0 in the same way as the right-ascension.
- Column 12: } The precession and secular variation in declination for 1900.0.
- Column 13: }
- Column 14: The proper motion in declination with its mean error.
- Column 15: Epoch of observation in declination.
- Column 16: The number of catalogues used for the determination of the proper motion.
- Column 17: The number of the star in the Bonner Durchmusterung.

¹ Lund Medd Ser. II No. 66 (1932).

Catalogue of the proper motions of 182

System: Ross PGC.

No.	Var.	Sp.	RA _{1900.0}	Precession	Var. saec.	μ_α	Epoch	Cat.
1	2	3	4	5	6	7	8	9
1	TT Peg	M ₃	0 ^h 1 ^m 20.802	+ 3.0763	+ .0160	— .0011 ± .0053	1895.45	3
2	ST Cas	Nb	12 14.334	3.1566	.0397	+ .0010 .0020	1916.47	5
3	VX And	N ₇	14 36.927	3.1550	.0331	— .0001 .0008	1914.18	8
4	T Cas	M8e	17 49.254	3.2219	.0499	+ .0035 .0000	1900.00	11
5	R And	Se	18 44.879	3.1577	.0275	— .0012 .0005	1904.80	12
6	VY Cas	M ₅	0 45 24.984	+ 3.5751	+ .0784	— .0182 ± .0038	1898.27	3
7	RT Psc	Mo	1 8 20.106	3.2690	.0215	— .0002 .0010	1901.22	5
8	Z Psc	No	10 37.690	3.2634	.0206	— .0021 .0004	1900.04	10
9	AA Cas	Mb	13 13.396	3.6901	.0640	— .0013 .0018	1902.47	3
10	SX And	Me	27 34.902	3.5885	.0445	— .0031 .0021	1912.32	3
11	U Per	M6e	1 52 56.193	+ 3.9534	+ .0646	+ .0006 ± .0010	1900.74	13
12	V Ari	R8	2 9 36.850	3.2216	.0134	+ .0010 .0005	1900.04	13
13	R Ari	M3e	10 25.595	3.4019	.0216	+ .0014 .0005	1890.45	14
14	T Per	K ₅	12 11.723	4.2617	.0797	— .0064 .0028	1910.11	3
15	AD Per	gK ₅	13 26.996	4.1841	.0723	— .0008 .0022	1912.68	5
16	RS Per	gM	2 15 20.357	+ 4.2031	+ .0729	+ .0024 ± .0025	1903.13	3
17	S Per	M ₅	15 40.532	4.2718	.0783	+ .0002 .0015	1914.98	6
18	R Tri	M6e	30 58.562	3.6206	.0293	— .0004 .0009	1914.81	8
19	YZ Per	M ₂	31 10.308	4.3150	.0723	— .0014 .0010	1915.79	6
20	Y Ari	M5e	35 1.865	3.5705	.0263	— .0010 .0007	1912.45	4
21	T Ari	M6	2 42 44.981	+ 3.3402	+ .0163	— .0018 ± .0005	1902.15	10
22	W Per	M9	43 14.736	4.3954	.0714	+ .0009 .0018	1914.04	7
23	Y Per	N _{3e}	3 20 54.348	4.0583	.0380	— .0016 .0013	1916.57	5
24	U Cam	N ₅	33 12.185	5.1153	.0865	+ .0012 .0013	1913.20	7
25	X Tau	F ₅	47 50.335	3.2193	.0101	+ .0033 .0004	1902.22	10
26	SW Per	M ₄	4 3 58.225	+ 4.1228	+ .0288	— .0029 ± .0013	1912.40	5
27	RV Cam	M ₃	22 23.336	4.9601	.0486	— .0034 .0026	1917.50	5
28	TT Tau	N ₃	45 15.057	3.7556	.0130	— .0008 .0007	1910.12	10
29	R Ori	Se	53 35.051	3.2518	.0067	— .0000 .0007	1902.55	6
30	W Ori	N ₅	5 0 13.838	3.0958	.0052	— .0010 .0003	1903.22	30
31	TX Aur	N ₃	5 2 13.185	+ 4.1156	+ .0147	— .0009 ± .0015	1919.28	5
32	UZ Aur	Ma	8 12.601	4.1659	.0139	+ .0004 .0007	1913.83	7
33	UV Aur	Se	15 17.954	3.9047	.0097	+ .0004 .0007	1912.42	8
34	S Aur	N ₃	20 30.745	3.9625	.0092	— .0021 .0008	1912.52	5
35	Y Aur	Gov	21 31.362	4.2736	.0117	— .0005 .0016	1912.24	3
36	CK Ori	Ko	5 25 2.841	+ 3.1677	+ .0041	— .0003 ± .0006	1890.73	15
37	RT Ori	Nb	27 49.969	3.2367	.0041	— .0046 .0007	1913.96	6
38	BN Ori	A ₇	31 6.321	3.2297	.0039	+ .0013 .0008	1875.02	5
39	TU Tau	N ₂	39 5.938	3.6754	.0045	+ .0002 .0006	1900.64	7
40	Y Tau	N ₂	39 41.832	3.5741	.0041	— .0015 .0005	1907.57	13
41	TW Aur	Mc	5 49 43.546	+ 4.4307	+ .0043	— .0014 ± .0010	1913.91	6
42	RS Aur	M ₃	56 26.599	4.4704	.0025	+ .0007 .0020	1916.50	3
43	TU Gem	N ₃	6 4 40.573	3.7250	.0009	— .0008 .0003	1902.46	24
44	TV Gem	Ma	5 50.462	3.6091	.0009	— .0006 .0006	1898.76	8
45	WY Gem	M3ep	5 51.646	3.6457	.0000	— .0018 .0004	1907.43	12

variable stars, determined at the Lund Observatory.

Precession: NEWCOMB.]

No.	Dec _{1900.0}	Precession	Var. saec.	μ_δ	Epoch	Cat.	BD
10	11	12	13	14	15	16	17
1	+ 26° 32' 0".26	+ 20".046	—".011	—".031 ± ".049	1894.57	3	+ 26° 47' 46"
2	49 43 54.11	20.018	.033	— .010 .020	1914.51	5	49 41
3	44 9 15.95	20.006	.037	— .008 .007	1908.25	8	43 53
4	55 14 17.39	19.986	.046	— .003 .008	1904.92	11	54 48
5	38 1 24.40	19.980	.047	— .019 .006	1900.19	12	37 58
6	+ 62 22 34.90	+ 19.654	— .111	— .105 ± .023	1896.40	3	+ 62 161
7	26 36 12.77	19.162	.149	— .050 .009	1898.46	5	26 199
8	25 14 27.35	19.102	.152	— .008 .005	1908.05	17	25 205
9	55 48 11.35	19.032	.177	+ .005 .014	1900.39	3	55 290
10	46 0 23.13	18.601	.204	+ .018 .024	1911.21	3	—
11	+ 54 20 9.44	+ 17.662	— .281	— .021 ± .010	1910.20	12	+ 54 431
12	11 46 19.21	16.925	.258	+ .045 .007	1906.07	13	11 305
13	24 35 30.17	16.887	.274	— .015 .007	1888.62	14	24 330
14	58 29 58.21	16.803	.346	— .013 .020	1908.88	3	58 439
15	56 31 58.18	16.743	.343	— .009 .011	1909.03	6	56 547
16	+ 56 39 6.49	+ 16.652	— .348	+ .014 ± .027	1903.19	2	+ 56 583
17	58 7 45.33	16.635	.356	+ .003 .012	1911.98	6	57 552
18	33 49 43.61	15.852	.330	— .021 .012	1913.36	8	33 470
19	56 36 41.88	15.842	.392	+ .006 .008	1915.12	6	56 673
20	30 46 12.82	15.633	.332	— .003 .010	1911.01	4	30 428
21	+ 17 5 30.83	+ 15.201	— .324	— .012 ± .007	1899.76	10	+ 16 351
22	56 34 4.56	15.173	.425	— .006 .013	1912.98	7	56 724
23	43 49 36.18	12.825	.460	— .011 .013	1914.24	6	43 726
24	62 19 24.15	11.980	.603	+ .006 .009	1909.82	7	62 596
25	7 28 30.20	10.930	.398	— .099 .006	1897.97	19	7 560
26	+ 41 56 50.94	+ 9.721	— .530	— .042 ± .014	1910.38	5	+ 41 824
27	57 11 27.21	8.282	.662	— .031 .018	1915.78	5	57 806
28	28 21 21.05	6.423	.521	+ .010 .009	1905.41	10	28 707
29	7 58 40.09	5.728	.457	— .037 .011	1895.07	6	7 768
30	1 2 24.35	5.169	.438	— .006 .004	1901.83	30	0 939
31	+ 38 52 17.78	+ 5.001	— .584	+ .007 ± .014	1916.21	5	+ 38 1035
32	40 0 57.72	4.492	.594	+ .055 .008	1910.55	8	39 1225
33	32 24 40.52	3.885	.560	— .029 .009	1912.73	8	32 957
34	34 3 43.88	3.437	.570	+ .003 .001	1911.00	5	34 1044
35	42 21 11.71	3.350	.616	— .020 .018	1910.24	3	42 1295
36	+ 4 7 37.83	+ 3.045	— .457	— .033 ± .009	1886.90	15	+ 4 949
37	7 4 55.42	2.804	.468	+ .000 .010	1913.25	6	7 929
38	6 46 13.17	2.521	.469	— .026 .012	1876.14	5	6 971
39	24 22 37.06	1.826	.534	— .007 .009	1908.49	7	24 943
40	20 39 10.99	1.774	.519	+ .011 .007	1906.39	13	20 1083
41	+ 45 29 37.98	+ 0.898	— .645	— .034 ± .010	1912.69	6	+ 45 1202
42	46 17 38.13	+ 0.311	.652	— .038 .022	1914.62	3	46 1089
43	26 2 2.11	— 0.409	.543	— .004 .004	1901.03	24	26 1117
44	21 53 22.48	— 0.511	.527	+ .003 .009	1895.64	8	21 1146
45	23 13 43.21	— 0.513	.532	— .004 .006	1906.95	10	23 1243

No.	Var.		Sp.	RA _{1900.0}	Precession	Var. saec.	μ_{α}	Epoch	Cat.
1	2		3	4	5	6	7	8	9
46	VW	Aur	M6	6 ^h 10 ^m 39 ^s .620	+ 3 ^s .9473	— .0005	+ .0043 ± .0022	1920.15	4
47	V	Lyn	—	20 28.415	5.5346	.0132	— .0032 .0080	1917.27	4
48	WW	Aur	A4sp	25 56.761	3.9191	.0034	— .0004 .0003	1898.88	25
49	RV	Aur	Na	27 35.800	4.2915	.0062	— .0037 .0018	1921.50	3
50	TU	Aur	Mb	28 14.306	4.4315	.0073	— .0002 .0010	1914.78	7
51	UU	Aur	N3	6 29 40.245	+ 4.1274	— .0055	+ .0006 ± .0004	1904.17	15
52	VW	Gem	Na	35 40.559	3.8830	.0049	— .0002 .0007	1914.51	9
53	RX	Gem	A2	43 38.327	3.9361	.0068	— .0000 .0008	1918.38	3
54	SX	Mon	Mb	46 38.996	3.1842	.0013	— .0021 .0009	1902.03	6
55	RV	Mon	Nb	53 0.080	3.2160	.0020	— .0021 .0009	1912.73	8
56	SW	Gem	Ko	6 53 19.328	+ 3.7117	— .0060	+ .0007 ± .0010	1907.90	6
57	R	Gem	Se	7 1 20.115	3.6157	.0062	+ .0002 .0004	1893.56	15
58	Y	Lyn	Mb	20 54.804	4.3796	.0228	+ .0005 .0010	1913.75	6
59	S	Gem	M4e	37 2.472	3.6068	.0105	— .0010 .0005	1907.00	9
60	W	CMi	R8	43 26.696	3.1911	.0046	— .0021 .0008	1911.09	6
61	RX	Cnc	M8	8 8 44.213	+ 3.6007	— .0142	— .0012 ± .0006	1906.49	19
62	V	Cnc	Se	16 1.056	3.4238	.0108	— .0007 .0005	1900.73	7
63	T	Lyn	Noe	16 22.406	3.8143	.0212	+ .0018 .0008	1914.50	5
64	Z	Cnc	M5	16 48.337	3.3750	.0097	— .0010 .0007	1917.89	5
65	RZ	Cnc	Ko	32 56.426	3.7320	.0213	— .0004 .0007	1907.78	3
66	T	Cnc	N3	8 50 57.479	+ 3.4341	— .0140	— .0005 ± .0004	1898.63	14
67	RT	Cnc	Ma	52 50.033	3.2658	.0092	— .0032 .0007	1911.94	8
68	TT	UMa	—	57 21.380	4.7745	.0837	+ .0005 .0015	1892.72	5
69	RS	Cnc	M6	9 4 36.346	3.6369	.0228	— .0008 .0004	1903.04	17
70	U	UMa	Map	10 8 13.794	4.1784	.0841	— .0002 .0006	1903.41	17
71	ST	UMa	Mb	11 22 22.567	+ 3.2964	— .0350	— .0027 ± .0007	1915.17	12
72	Z	UMa	M6e	51 17.370	3.1550	— .0498	— .0047 .0014	1920.23	9
73	TZ	Vir	Ma	59 28.805	3.0725	+ .0002	+ .0008 .0013	1897.86	6
74	TZ	UMa	Ma	12 4 34.863	3.0280	— .0460	+ .0013 .0020	1909.10	5
75	Y	CVn	N3	40 25.917	2.8296	— .0214	— .0004 .0003	1900.16	40
76	RY	Dra	N4p	12 52 29.947	+ 2.3732	— .0315	+ .0003 ± .0045	1916.43	6
77	RT	Vir	M7	57 33.715	3.0390	+ .0021	+ .0013 .0005	1915.65	13
78	RS	CVn	F4n	13 5 58.687	2.7919	— .0118	— .0042 .0010	1922.19	6
79	UW	UMa	—	7 54.989	2.4734	— .0210	+ .0109 .0022	1907.49	4
80	V	UMi	M4	36 53.682	1.0517	+ .0392	+ .0084 .0017	1899.74	8
81	R	CVn	M7e	13 44 39.722	+ 2.5771	— .0080	— .0003 ± .0007	1914.38	9
82	RW	CVn	—	55 16.095	2.5747	— .0061	+ .0018 .0010	1902.78	6
83	W	CVn	F1v	14 2 14.896	2.5356	— .0055	— .0018 .0010	1912.12	5
84	Y	Boo	Ko	17 22.168	2.7940	+ .0006	— .0015 .0004	1910.84	18
85	UV	Boo	F5	18 2.457	2.7029	— .0010	— .0012 .0009	1899.04	8
86	RX	Boo	M8	14 19 41.881	+ 2.6965	— .0009	+ .0001 ± .0004	1911.51	17
87	V	Boo	M6e	25 43.012	2.4226	— .0029	+ .0022 .0008	1914.78	10
88	RV	Boo	M7e	35 2.814	2.5296	— .0014	+ .0007 .0006	1918.72	9
89	WR	Boo	Mb	36 58.985	2.5440	— .0010	— .0064 .0006	1918.49	12
90	U	CrB	B3	15 14 6.725	2.4463	+ .0013	— .0017 .0006	1912.56	8
91	RU	CrB	M7	15 31 21.460	+ 2.5509	+ .0025	— .0049 ± .0011	1910.03	5
92	RR	CrB	Mb	37 46.386	2.1957	.0024	+ .0002 .0005	1909.10	13
93	V	CrB	N3e	45 56.983	2.1414	.0028	+ .0007 .0006	1907.45	10
94	ST	Her	Mc	47 47.036	1.7935	.0052	— .0006 .0009	1913.95	10
95	RS	CrB	Mb	54 47.677	2.2333	.0028	— .0037 .0010	1919.25	8

No.	Dec _{1900.0}	Precession	Var. saec.	μ_δ	Epoch	Cat.	BD
10	11	12	13	14	15	16	17
46	+ 33° 14' 25".09	— 0".932	—".573	+ ".008 ± ".027	1920.28	4	+ 33° 1290
47	61 36 12.94	1.788	.803	— .000 .059	1915.90	4	61 887
48	32 31 33.90	2.265	.569	— .025 .004	1890.89	24	32 1324
49	42 34 44.77	2.408	.620	— .026 .020	1921.85	3	42 1571
50	45 42 2.38	2.464	.639	— .023 .010	1912.63	7	45 1324
51	+ 38 31 34.27	— 2.588	— .596	— .026 ± .004	1902.43	18	+ 38 1539
52	31 32 57.02	3.108	.558	— .032 .009	1913.57	9	31 1388
53	33 21 8.94	3.794	.562	— .027 .012	1918.30	3	33 1415
54	4 53 0.82	4.052	.454	— .013 .012	1899.33	7	4 1476
55	6 18 2.22	4.595	.455	+ .006 .011	1911.02	8	6 1462
56	+ 26 10 48.63	— 4.622	— .526	— .010 ± .012	1904.19	6	+ 26 1412
57	22 51 28.30	5.301	.507	— .003 .005	1886.66	16	22 1577
58	46 11 29.84	6.931	.596	— .010 .011	1910.77	6	46 1271
59	23 41 6.27	8.237	.478	— .020 .006	1900.96	9	23 1796
60	5 38 30.50	8.744	.414	— .011 .012	1908.73	6	5 1797
61	+ 25 2 16.13	— 10.678	— .440	— .022 ± .007	1904.12	19	+ 25 1880
62	17 36 8.05	11.211	.409	— .009 .008	1899.21	7	17 1825
63	33 50 16.40	11.237	.457	+ .005 .011	1915.07	5	33 1686
64	15 18 41.21	11.268	.402	— .051 .012	1919.80	5	15 1808
65	32 8 47.94	12.407	.421	+ .000 .010	1912.54	3	32 1772
66	+ 20 13 54.98	— 13.605	— .365	— .003 ± .005	1893.09	13	+ 20 2243
67	11 13 55.57	13.725	.341	— .031 .008	1907.60	8	11 1954
68	60 40 59.19	14.011	.492	— .010 .011	1892.98	5	60 1169
69	31 22 24.48	14.457	.361	— .033 .005	1900.99	15	31 1946
70	60 28 52.10	17.710	.278	+ .011 .004	1902.56	17	60 1246
71	+ 45 44 9.66	— 19.777	— .070	— .035 ± .006	1911.80	13	+ 45 1924
72	58 25 42.09	20.032	— .010	— .008 .010	1919.35	9	58 1346
73	3 10 36.22	20.047	+ .008	— .004 .019	1897.38	6	3 2593
74	58 56 26.81	20.043	+ .017	— .010 .014	1908.06	5	59 1423
75	45 59 13.02	19.736	+ .080	+ .019 .003	1899.91	40	46 1817
76	+ 66 32 8.27	— 19.523	+ .087	+ .030 ± .027	1916.24	6	+ 66 780
77	5 43 26.03	19.418	.119	— .016 .008	1915.08	12	5 2708
78	36 28 0.02	19.222	.124	+ .000 .010	1920.40	6	36 2344
79	56 54 40.49	19.173	.113	— .004 .015	1905.12	4	57 1422
80	74 49 1.74	18.282	.071	+ .002 .008	1904.51	9	75 512
81	+ 40 2 24.24	— 17.992	+ .175	— .008 ± .008	1913.45	10	+ 40 2694
82	37 40 59.64	17.564	.188	— .019 .010	1898.18	6	37 2480
83	38 18 18.96	17.262	.196	— .005 .009	1923.56	5	38 2514
84	20 15 45.35	16.552	.237	— .029 .005	1908.37	18	20 2970
85	26 0 22.32	16.519	.231	— .019 .010	1897.63	8	26 2559
86	+ 26 9 30.14	— 16.437	+ .233	— .035 ± .005	1908.45	17	+ 26 2563
87	39 18 24.68	16.129	.216	— .052 .008	1911.82	10	39 2773
88	32 58 13.75	15.632	.238	— .012 .008	1918.71	9	32 2482
89	31 59 59.71	15.525	.241	— .022 .008	1917.92	12	32 2504
90	32 0 44.26	13.276	.273	— .024 .007	1909.35	8	32 2569
91	+ 26 5 6.45	— 12.109	+ .302	— .002 ± .014	1908.71	5	+ 26 2704
92	38 52 42.01	11.657	.265	— .046 .006	1911.20	12	39 2901
93	39 52 32.57	11.068	.264	— .031 .007	1907.73	10	40 2929
94	48 47 3.16	10.934	.224	+ .007 .009	1913.04	11	48 2334
95	36 18 27.18	10.415	.283	— .008 .011	1917.48	8	36 2672

No.	Var.		Sp.	RA _{1900.0}	Precession	Var. saec.	μ_{α}	Epoch	Cat.
1	2		3	4	5	6	7	8	9
96	X	Her	Mc	15 ^h 59 ^m 38 ^s .627	+ 1.8098	+ .0051	— .0084 ± .0009	1917.92	10
97	BE	Her	M ₃	16 21 6.446	2.3861	.0032	— .0064 .0009	1915.32	6
98	UY	Her	A ₂	29 41.682	2.0984	.0036	— .0012 .0009	1919.11	7
99	W	Her	M _{3e}	31 40.158	2.1204	.0036	— .0012 .0008	1915.39	5
100	UU	Her	cF8	32 27.412	2.0975	.0036	— .0022 .0013	1919.82	6
101	UW	Her	M _{4e}	17 10 53.654	+ 2.1067	+ .0033	— .0002 ± .0008	1918.60	9
102	VW	Dra	Ko	15 16.583	0.7287	.0108	— .0060 .0006	1901.00	19
103	TX	Her	A ₂	15 27.129	1.8920	.0036	— .0013 .0007	1913.90	7
104	AV	Oph	—	18 3 40.939	2.9494	.0017	— .0010 .0013	1888.62	6
105	W	Lyr	M _{4e}	11 27.729	2.0797	.0020	+ .0012 .0012	1919.53	5
106	TZ	Dra	—	18 19 29.917	+ 1.6182	+ .0015	— .0016 ± .0011	1902.63	6
107	T	Lyr	N ₃	28 52.446	2.0760	+ .0017	— .0019 .0012	1918.36	6
108	XY	Lyr	Mb	34 48.338	1.9802	+ .0014	+ .0002 .0004	1896.87	21
109	T	Aql	Mb	40 56.527	2.8725	+ .0004	+ .0011 .0015	1887.14	7
110	UW	Aql	Mo	52 27.094	3.0650	— .0010	— .0006 .0009	1907.61	6
111	UV	Aql	N ₄	18 53 58.088	+ 2.7428	+ .0005	— .0004 ± .0007	1916.56	7
112	RT	Vul	Ao	19 7 13.731	2.5498	+ .0008	— .0030 .0006	1902.70	5
113	CH	Cyg	M ₄	21 55.368	1.5779	— .0014	— .0010 .0004	1900.72	23
114	RR	Lyr	A _{5.5v}	22 16.648	1.9222	+ .0005	— .0115 .0007	1911.92	7
115	AW	Cyg	N ₃	25 48.382	1.7916	.0000	— .0013 .0012	1917.93	5
116	AF	Cyg	M ₄	19 27 13.363	+ 1.7902	— .0002	+ .0021 ± .0007	1905.50	7
117	UV	Cyg	Mc?	28 3.222	1.8997	+ .0003	— .0050 .0021	1922.17	2
118	RT	Aql	M _{7e}	33 19.234	2.8229	— .0008	— .0004 .0022	1922.65	2
119	R	Cyg	S	34 8.121	1.6134	— .0016	— .0007 .0005	1894.81	19
120	TT	Cyg	N ₃	37 6.980	2.2997	+ .0012	— .0014 .0007	1914.19	6
121	S	Vul	cK7v	19 44 17.787	+ 2.4597	+ .0010	— .0021 ± .0004	1909.67	14
122	χ	Cyg	M _{6pe}	46 43.298	2.3068	.0013	— .0027 .0004	1900.56	12
123	SV	Vul	K1v	47 24.936	2.4594	.0010	— .0029 .0005	1914.28	10
124	AX	Cyg	Nb	53 58.685	1.9384	.0005	— .0016 .0005	1912.20	10
125	QZ	Cyg	M ₃	55 28.191	2.1583	.0013	— .0058 .0013	1912.60	3
126	AH	Cyg	—	19 57 6.951	+ 2.0975	+ .0012	— .0030 ± .0011	1919.57	4
127	X	Sge	Nb	20 0 40.487	2.6435	.0004	— .0016 .0006	1908.86	6
128	AA	Cyg	S	0 45.943	2.2166	.0015	— .0030 .0012	1921.01	6
129	W	Vul	M _{4e}	5 52.491	2.5166	.0011	— .0021 .0017	1924.22	3
130	AY	Cyg	N	6 16.664	2.0755	.0014	— .0036 .0014	1921.38	4
131	SV	Cyg	N ₃	20 6 27.562	+ 1.8270	— .0001	— .0075 ± .0028	1918.04	4
132	RY	Cyg	Nb	6 37.547	2.2564	+ .0017	+ .0040 .0019	1923.24	4
133	R	Sge	cG7v	9 29.728	2.7396	— .0002	— .0028 .0004	1899.23	13
134	RS	Cyg	Nope	9 45.761	2.1775	+ .0016	— .0007 .0006	1908.42	10
135	AC	Cyg	Mc	9 52.839	1.7685	— .0010	+ .0010 .0011	1918.21	3
136	U	Cyg	Npe	20 16 30.385	+ 1.8617	+ .0002	— .0011 ± .0008	1912.46	6
137	RS	Del	Mc	24 32.881	2.7640	— .0002	+ .0013 .0006	1900.36	6
138	RW	Cyg	M ₁	25 12.574	2.1798	+ .0021	— .0038 .0008	1918.08	9
139	AD	Cyg	S	27 36.568	2.3986	+ .0023	— .0012 .0006	1914.86	7
140	AI	Cyg	M ₄	27 44.924	2.4001	+ .0023	— .0008 .0009	1912.83	4
141	V	Vul	cG _{5pv}	20 32 16.662	+ 2.5533	+ .0017	— .0038 ± .0007	1913.88	8
142	S	Del	M ₅	38 28.428	2.7629	.0003	+ .0002 .0005	1902.31	9
143	X	Cyg	G _{4.5v}	39 29.255	2.3480	.0030	— .0013 .0004	1900.13	16
144	RR	Cyg	M?	42 36.909	2.0758	.0026	— .0057 .0015	1916.84	5
145	Y	Cyg	O _{9nnk}	48 3.733	2.3955	.0033	+ .0005 .0008	1900.79	5

No.	Dec _{1900.0}	Precession	Var. saec.	μ_δ	Epoch	Cat.	BD
10	11	12	13	14	15	16	17
96	+ 47° 30' 52".35	- 10".050	+ ".233	+ ".056 ± ".010	1917.53	9	+ 47° 2291
97	29 28 47.11	8.384	.319	-.017 .010	1911.92	6	29 2824
98	38 16 52.10	7.696	.286	-.026 .007	1913.77	8	38 2791
99	37 33 1.07	7.537	.290	-.003 .010	1915.27	5	37 2771
100	38 10 15.22	7.473	.287	-.034 .012	1918.70	6	38 2803
101	+ 36 28 56.41	- 4.263	+ .302	-.012 ± .009	1916.27	9	+ 36 2843
102	60 46 36.04	- 3.887	.106	+ .018 .003	1896.39	20	60 1743
103	41 59 30.60	- 3.872	.272	-.019 .008	1912.16	7	42 2823
104	5 15 30.12	+ 0.322	.430	+ .001 .019	1888.52	6	5 3615?
105	36 38 18.26	+ 1.002	.303	-.022 .012	1917.88	5	36 3066
106	+ 47 31 2.63	+ 1.703	+ .234	+ .027 ± .010	1898.16	6	+ 47 2625
107	36 55 28.66	2.519	.299	+ .002 .011	1915.81	6	36 3168
108	39 34 47.07	3.033	.284	-.006 .004	1890.57	21	39 3476
109	8 38 18.80	3.562	.411	-.012 .021	1887.31	6	8 3835
110	0 19 23.30	4.548	.434	+ .001 .013	1906.66	6	0 4064
111	+ 14 13 42.29	+ 4.677	+ .387	-.006 ± .010	1910.75	6	+ 14 3729
112	22 13 1.87	5.796	.354	-.005 .009	1899.92	5	22 3617
113	50 2 38.57	7.014	.213	-.015 .003	1897.22	22	49 2999
114	42 35 25.04	7.042	.259	-.198 .008	1911.37	7	42 3338
115	45 50 12.14	7.331	.240	+ .043 .012	1907.06	5	45 2906
116	+ 45 56 15.38	+ 7.447	+ .239	-.005 ± .007	1898.59	8	+ 45 2913
117	43 25 30.65	7.514	.253	-.063 .023	1918.67	2	43 3268
118	11 29 44.96	7.939	.375	+ .039 .039	1922.30	2	11 3919 ^a
119	49 58 29.08	8.004	.212	-.004 .005	1891.39	16	49 3064
120	32 23 6.20	8.243	.301	-.000 .009	1913.51	6	32 3522
121	+ 27 2 14.73	+ 8.811	+ .319	-.011 ± .005	1903.86	14	+ 26 3674
122	32 39 39.20	9.001	.297	-.051 .005	1896.15	12	32 3593
123	27 12 15.71	9.055	.316	-.009 .006	1909.61	11	27 3536
124	43 59 31.87	9.564	.245	-.010 .006	1908.97	10	43 3425
125	37 59 22.02	9.678	.271	-.018 .013	1907.49	3	37 3710
126	+ 39 54 15.31	+ 9.804	+ .263	+ .022 ± .013	1916.50	4	+ 39 3997
127	20 21 49.10	10.074	.329	-.036 .009	1905.30	7	20 4417
128	36 31 58.75	10.081	.276	-.023 .013	1919.30	6	36 3852
129	25 59 26.04	10.465	.309	-.030 .015	1920.34	3	25 4126
130	41 11 55.62	10.495	.253	-.054 .017	1919.75	4	41 3632
131	+ 47 34 34.64	+ 10.508	+ .222	+ .020 ± .026	1916.21	4	+ 47 3031
132	35 39 2.37	10.521	.276	+ .029 .027	1922.30	4	35 4002
133	16 25 24.86	10.734	.333	+ .003 .005	1894.79	14	16 4197
134	38 25 35.48	10.753	.263	+ .001 .006	1905.99	10	38 3957
135	49 8 53.33	10.762	.209	+ .013 .011	1914.26	3	49 3225
136	+ 47 34 42.97	+ 11.247	+ .220	-.009 ± .009	1909.54	6	+ 47 3077
137	15 56 26.98	11.822	.321	+ .018 .009	1892.81	6	15 4172
138	39 38 55.34	11.869	.252	-.006 .010	1916.79	9	39 4208
139	32 13 35.04	12.037	.275	-.010 .009	1913.90	7	32 3850
140	32 11 1.62	12.047	.275	-.027 .012	1910.83	4	32 3852
141	+ 26 15 23.86	+ 12.361	+ .289	-.013 ± .009	1911.34	8	+ 26 3937
142	16 43 42.28	12.783	.305	+ .014 .008	1898.62	8	16 4351
143	35 13 37.93	12.851	.258	-.001 .005	1895.48	15	35 4234
144	44 30 12.06	13.060	.223	-.023 .016	1914.77	5	44 3571 ^a
145	34 16 54.14	13.418	.255	-.007 .010	1892.39	6	34 4184

No.	Var.		Sp.	RA _{1900.0}	Precession	Var. saec.	μ_α	Epoch	Cat.
1	2		3	4	5	6	7	8	9
146	AZ	Cyg	Map	20 ^h 54 ^m 32 ^s .488	+ 2 ^s .0679	+ ^s .0033	— ^s .0012 ± ^s .0007	1915.36	10
147	YZ	Cyg	Ao	58 54.614	2.2501	+ .0042	— .0017 .0012	1909.76	3
148	T	Cep	M6e	21 8 13.027	0.8093	— .0293	— .0033 .0008	1881.27	8
149	YY	Cyg	Nc	18 39.052	2.2943	+ .0059	— .0022 .0011	1917.83	5
150	SW	Cep	Mc	23 18.722	1.4753	— .0055	+ .0007 .0027	1918.40	5
151	W	Cyg	M4e	21 32 14.378	+ 2.2712	+ .0073	+ .0054 ± .0007	1902.51	12
152	AB	Cyg	M4e	32 15.518	2.5772	.0058	+ .0016 .0006	1912.68	10
153	UU	Cyg	A2	35 33.482	2.3423	.0078	— .0012 .0008	1905.50	7
154	RU	Cyg	M8e	37 18.439	2.0047	.0062	— .0051 .0020	1918.32	4
155	RV	Cyg	N5	39 7.978	2.4797	.0075	+ .0004 .0006	1911.46	11
156	TU	Peg	Me	21 40 13.905	+ 2.9063	+ .0005	— .0005 ± .0010	1919.94	2
157	μ	Cep	M2	40 26.784	1.8336	.0040	— .0007 .0005	1905.51	24
158	VZ	Cyg	G2v	47 41.009	2.4000	.0092	— .0065 .0012	1903.24	4
159	LW	Cyg	Nc	51 30.724	2.2247	.0099	+ .0003 .0009	1918.51	5
160	RX	Peg	N3	51 44.006	2.7801	.0046	— .0007 .0005	1909.52	8
161	UV	Peg	Ao	21 59 47.840	+ 2.5989	+ .0090	— .0009 ± .0008	1911.07	6
162	SV	Peg	Mc	22 1 20.361	2.6116	+ .0091	— .0000 .0008	1914.16	4
163	UW	Peg	M4	13 5.901	3.0489	— .0024	— .0012 .0008	1922.16	3
164	TX	Peg	M5	13 24.784	2.9328	+ .0022	+ .0003 .0008	1913.85	6
165	RW	Cep	Ma	19 21.568	2.2469	+ .0156	— .0000 .0006	1889.20	18
166	ST	Cep	Mo	22 26 22.905	+ 2.2706	+ .0173	— .0027 ± .0015	1922.82	5
167	W	Cep	Pec	32 38.688	2.2795	.0189	— .0015 .0016	1917.51	3
168	BD	Peg	M8	38 15.339	2.8280	.0100	— .0001 .0007	1917.52	6
169	UY	Peg	M1	40 2.983	2.8111	.0111	— .0004 .0013	1912.67	3
170	U	Lac	M4	43 36.790	2.4564	.0219	— .0051 .0020	1921.14	4
171	ST	Peg	M6e	22 44 18.729	+ 2.8531	+ .0102	— .0013 ± .0008	1915.19	7
172	RX	Lac	M5	45 25.228	2.7071	.0164	+ .0007 .0012	1922.86	2
173	SX	Lac	K2	51 19.325	2.7995	.0145	— .0009 .0007	1916.52	4
174	TV	Lac	Nb	51 52.174	2.5395	.0239	+ .0032 .0013	1916.11	5
175	TV	And	M4	53 29.579	2.7256	.0184	— .0019 .0015	1920.78	4
176	R	Peg	M7e	23 1 37.583	+ 3.0129	+ .0036	— .0009 ± .0005	1901.42	8
177	RZ	And	Mo	5 2.897	2.6587	.0267	— .0079 .0020	1923.22	3
178	TZ	And	M5	45 51.760	2.9841	.0312	— .0004 .0016	1919.64	5
179	TZ	Cas	M2	47 58.897	2.9488	.0496	— .0020 .0013	1912.80	8
180	RS	And	Mc	50 18.992	3.0095	.0331	— .0005 .0013	1919.59	5
181	R	Cas	M7e	23 53 19.629	+ 3.0246	+ .0369	+ .0081 ± .0005	1894.50	14
182	SU	And	Nb	59 27.980	3.0694	.0296	— .0004 .0011	1911.82	6

No.	Dec _{1900.0}	Precession	Var. saec.	μ_δ	Epoch	Cat.	BD
10	11	12	13	14	15	16	17
146	+ 46° 4' 46".49	+ 13".834	+ ".213	— ".009 ± ".007	1912.76	9	+ 45° 3349
147	40 53 27.08	14.108	.227	— .029 .014	1905.10	3	40 4393
148	68 4 59.65	14.674	.074	— .045 .009	1905.09	8	67 1291
149	41 58 7.57	15.281	.210	— .004 .011	1916.52	5	41 4114
150	62 8 25.32	15.541	.129	+ .003 .017	1916.45	5	61 2134
151	+ 44 55 36.01	+ 16.023	+ .192	— .014 ± .008	1897.19	12	+ 44 3877
152	31 39 13.53	16.024	.220	+ .009 .008	1910.82	10	31 4504
153	42 49 31.72	16.195	.194	— .008 .009	1901.48	7	42 4172
154	53 52 12.89	16.285	.164	— .013 .017	1917.14	4	53 2684
155	37 33 35.19	16.379	.202	— .005 .006	1907.38	11	37 4407
156	+ 12 14 19.61	+ 16.433	+ .236	— .028 ± .013	1907.75	2	+ 12 4678
157	58 19 16.58	16.444	.146	— .009 .003	1901.18	25	58 2316
158	42 39 53.36	16.797	.184	— .002 .014	1899.58	4	42 4233
159	50 1 23.52	16.978	.165	— .005 .009	1916.92	5	49 3673
160	22 23 11.98	16.988	.208	+ .003 .007	1906.56	8	22 4508
161	+ 35 16 30.13	+ 17.352	+ .183	— .013 ± .009	1909.20	5	+ 35 4699
162	34 51 43.58	17.419	.181	+ .008 .008	1907.98	4	34 4597
163	2 13 48.77	17.905	.192	+ .010 .013	1920.22	3	2 4488
164	13 6 30.06	17.918	.185	— .061 .010	1909.16	6	12 4801
165	55 27 26.39	18.145	.131	+ .006 .005	1876.53	20	55 2737
166	+ 56 29 17.31	+ 18.397	+ .124	+ .000 ± .009	1918.47	6	+ 56 2793
167	57 54 27.15	18.608	.116	+ .002 .012	1910.28	3	57 2568
168	27 38 0.42	18.785	.136	— .007 .008	1913.24	6	27 4389
169	29 46 2.51	18.839	.132	— .003 .014	1907.18	3	29 4745
170	54 37 51.96	18.944	.109	— .049 .018	1920.44	4	54 2863
171	+ 26 49 44.75	+ 18.963	+ .127	— .011 ± .009	1909.66	7	+ 26 4507
172	40 31 17.57	18.995	.119	+ .002 .012	1917.06	2	40 4920
173	34 39 43.03	19.153	.112	— .028 .009	1915.22	4	34 4794
174	53 41 43.82	19.167	.100	— .017 .013	1916.08	5	53 3033
175	42 12 4.63	19.209	.107	— .009 .017	1919.47	4	41 4651
176	+ 10 0 11.59	+ 19.400	+ .103	— .031 ± .007	1897.78	8	+ 9 5158
177	52 30 8.14	19.473	.084	+ .009 .020	1922.80	3	52 3375
178	46 57 5.22	20.009	.018	+ .010 .016	1918.18	5	46 4187
179	60 26 45.68	20.019	.014	— .013 .011	1911.79	7	60 2634
180	48 4 54.63	20.029	.010	+ .005 .013	1918.18	5	47 4318
181	+ 50 49 53.30	+ 20.038	+ .005	+ .008 ± .005	1892.12	14	+ 50 4202
182	42 59 39.85	20.047	— .007	+ .005 .011	1910.51	6	42 4827

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